THE
Young Algebraiſt's Companion:
OR,
A NEW and EASY GUIDE to
ALGEBRA;
Introduced by the Doctrine of
VULGAR FRACTIONS:
Deſigned for the Uſe of SCHOOLS, and ſuch who, by their own Application only, would become acquainted with the Rudiments of this noble Science:

Illuſtrated with

Variety of numerical and literal Examples, and attempted in natural and familiar Dialogues, in order to render the Work more eaſy and diverting to thoſe that are quite unacquainted with *Fractions* and the *Analytic* Art.

WITH
AN APPENDIX
On the Rudiments of Quadratic Equations, with ſeveral practical PROBLEMS, ſetting forth the Manner of compleating the Square, and extracting the Roots of Powers: Alſo new and eaſy geometrical Definition of the Difference between the ſolid Content of the Cylinder, and the Parallelopiped proved by the Pen.

BY DANIEL FENNING,

Late of the Royal Exchange Aſſurance Office, and Author of the Royal Engliſh Dictionary; The Britiſh Youth's Inſtructor; The Young Man's Book of Knowledge; The Ready Reckoner, being correct Tables ready caſt up; and A New Grammar of the Engliſh Language.

THE FIFTH EDITION.

To which is now added,
A SUPPLEMENT,
Containing the Methods of exterminating two, three, or more unknown Quantities out of an Equation; ſome further Obſervations on Quadratics, &c.

By the Rev. JOHN HELLINS,

Of Green's-Norton, in Northamptonſhire; formerly an *Aſtronomical Obſerver* at *Greenwich*.

LONDON:
Printed for S. CROWDER, Pater-noſter-Row, 1787.

BRITISH MUSEUM

ADVERTISEMENT

To this FIFTH EDITION.

THE Sale of four numerous Editions of this Book ſince the Year 1750, when it was firſt publiſhed, is no ſmall Proof of its Utility. And the preſent Demand for the YOUNG ALGEBRAIST'S COMPANION, notwithſtanding ſome late Algebraic Publications, ſufficiently ſhews, that the Public are the beſt Judges of Merit, and that the Effect of pompous Advertiſements and partial Accounts of Books, is but of ſhort Duration.

The Proprietors of this Work, in Gratitude for the Encouragement which they have received from the Public, have procured this fifth Edition to be printed under the Inſpection of a Gentleman of known Abilities in the Mathematical Sciences, by whom the Book has been corrected in ſeveral Places, and enlarged by the Addition of Notes and the Supplement.

To the AUTHOR of the YOUNG ALGEBRAIST's COMPANION, *&c.*

SIR,

THERE is nothing can give a greater Satiſfaction to a Lover of the Sciences, than to ſee them handled in a clear and maſterly Manner; and that every Attempt to remove Difficulties, and clear up the Obſcurities of any knotty Part of Science (ſo as to level it to the Capacities of Youth, and at the ſame Time to make it pleaſing as well as inſtructing) muſt be allowed, by *all*, to be a difficult Taſk. Give me Leave therefore to thank you for the laudable Pains you have taken in your YOUNG ALGEBRAIST's COMPANION, wherein you have made *that* which was hard and difficult, plain and eaſy to be underſtood, and at the ſame Time, have wrote it in ſo engaging a Manner, that the moſt inattentive Peruſer muſt receive at once both Pleaſure and Information. I am,

SIR,

Your unknown Friend,

TOWER-ROYAL,
London, Aug. 21, 1750.

and humble Servant,

SAMUEL HILL,
Philom.

N. B. *The* Author *has had ſeveral ſuch friendly Letters as this, from other Strangers in the* City *and* Country.

To the Reverend Mr. HELLINS, *at* GREEN'S NORTON, NORTHAMPTONSHIRE.

DEAR SIR,

I Am exceedingly glad to hear, that the late worthy Mr. FENNING's *Young Algebraiſt's Companion* is once more coming from the Preſs, it having been long conſidered as a moſt facile Introduction to Algebra, that noble and moſt comprehenſive Branch of the Mathematics. I cannot but congratulate thoſe who are deſirous of entering on theſe Studies, upon the Appearance of a new and improved Edition of this uſeful little Book, eſpecially as it comes from a Hand ſo capable of giving it every neceſſary Advantage.—From this Conſideration, and my own Experience, I can with Pleaſure moſt heartily recommend it to all Learners, particularly thoſe who have not the Aſſiſtance of a Tutor, as the eaſieſt and beſt Book of its Kind extant.—With a truly grateful Senſe of the great Service you have done to Science in this Work as an Editor,

I am, Rev. Sir,

Your moſt reſpectful Friend,

and humble Servant,

T. WOOLSTON.

Boarding-School,
Adderbury, Oxfordſh.
Nov. 21, 1786.

☞ The Editor has received a Letter to the ſame Effect from Mr. JOHN JEWELL, Teacher of the Mathematics at *Bideford*, Devon.

To the PROPRIETORS of FENNING'S ALGEBRA.

SIRS,

I Have not only attentively peruſed each Sheet of your *Young Algebraiſt's Companion*, but have likewiſe carefully examined every Operation; and being by theſe Means perfectly acquainted with the many *Corrections*, *Additions*, and *Improvements*, made by the Rev. Mr. HELLINS (the preſent Editor), I can with Confidence recommend this Fifth Edition as a good Introduction to the Knowledge of *Vulgar Fractions* and *Algebra*.

JOSEPH MOON,
Philomath.

SALISBURY,
March 17, 1787.

WE whoſe Names are here under-written, having each of us peruſed the following Sheets, do allow the Dialogues and Demonſtrations to be very natural, and eaſily adapted; and therefore beg Leave to recommend the Work, as one of the plaineſt and beſt ſuited to the Capacities of young Beginners extant.

ANTHONY GILBERT.
PETER DENNIS, Surveyor.
SAMUEL HILL, Philom.
THOMAS HUMPHREYS.
TIMOTHY LANGLEY, Accomptant.
JOSEPH SIMPSON, Teacher of the Mathematics.
JOHN REPTON, Superviſor.
ABRAHAM DE LIRE, Teacher of the Mathematics.
JOHN QUANT, Writing-Maſter and Accomptant.
GEORGE COLES, Land Surveyor.
RICHARD RICHARDSON.
ERASMUS TURNER.
ZACHARIAH SNAPER, Accomptant.
T. WOOLSTON, Maſter of a Boarding-School, Adderbury, Oxfordſhire.
JOHN JEWELL, Teacher of the Mathematics, at Bideford, Devon.
JOS. MOON, Teacher of the Mathematics, Saliſbury.
R. M. JONES, of Norton, School-Maſter, and Land-Surveyor.

To the PROPRIETORS of FENNING'S ALGEBRA.

SIRS,

I Have not only attentively peruſed each Sheet of your *Young Algebraiſt's Companion*, but have likewiſe carefully examined every Operation; and being by theſe Means perfectly acquainted with the many *Corrections*, *Additions*, and *Improvements*, made by the Rev. Mr. HELLINS (the preſent Editor), I can with Confidence recommend this Fifth Edition as a good Introduction to the Knowledge of *Vulgar Fractions* and *Algebra*.

JOSEPH MOON,
Philomath.

SALISBURY,
March 17, 1787.

WE whoſe Names are here under-written, having each of us peruſed the following Sheets, do allow the Dialogues and Demonſtrations to be very natural, and eaſily adapted; and therefore beg Leave to recommend the Work, as one of the plaineſt and beſt ſuited to the Capacities of young Beginners extant.

ANTHONY GILBERT.
PETER DENNIS, Surveyor.
SAMUEL HILL, Philom.
THOMAS HUMPHREYS.
TIMOTHY LANGLEY, Accomptant.
JOSEPH SIMPSON, Teacher of the Mathematics.
JOHN REPTON, Superviſor.
ABRAHAM DE LIRE, Teacher of the Mathematics.
JOHN QUANT, Writing-Maſter and Accomptant.
GEORGE COLES, Land Surveyor.
RICHARD RICHARDSON.
ERASMUS TURNER.
ZACHARIAH SNAPER, Accomptant.
T. WOOLSTON, Maſter of a Boarding-School, Adderbury, Oxfordſhire.
JOHN JEWELL, Teacher of the Mathematics, at Bideford, Devon.
JOS. MOON, Teacher of the Mathematics, Saliſbury.
R. M. JONES, of Norton, School-Maſter, and Land-Surveyor.

DEDICATION.

To the Honourable the Governors and Directors of the Royal Exchange Aſſurance Company.

GENTLEMEN,

IT being not only a common, but a very juſt Obſervation, that the Generality of Dedications are carried to too high an Extreme of flattering Encomiums, I perſuade myſelf you will the more readily accept of one in a *plain* Dreſs only, as I am ſenſible it is no Ways agreeable to your *Honours* to be flattered.

When ſolicited by ſeveral Gentlemen in Town and Country to publiſh the following Tract, I was not long conſidering at whoſe Feet to lay it: The Preſumption indeed was ſo great, that though the Thoughts of former Favours in ſome Meaſure encouraged me, yet it was with Reluctance I preſumed to beg *this*; and your permitting me to ſend it abroad under ſuch Protection, makes it difficult to ſay which is greater, the Honour or the Kindneſs you have done me, ſince from the *One* I ſhall naturally reap the Be-Benefit of the *Other*.

Even Books of *common Arithmetic* only have ſeldom wanted Patronizers, but Gentlemen of the firſt Claſs have more eſpecially condeſcended to protect and maintain the more ſuperior Parts of *Mathematical* Learning, not only from the Pleaſure and Satisfaction that ariſes even from the very *ſpeculative* Knowledge of them, but by being ſo uſeful in exerciſing the Faculties, and bringing the Mind into a juſt and proper Method of Reaſoning, were always thought worthy the Study of the beſt of Men, are eſteemed as pretty Decorations and Accompanyments to other Branches of Learning, and for Want of which, Education is always counted ſomething the leſs complete.

It is ſuperfluous indeed to mention this, ſince it is evident your *Honours* were ſenſible of the Truth of it, from your Readineſs to promote a Work wrote by one who at beſt profeſſes himſelf but a *Novice* in the thorough Knowledge of this extenſive Study: So that your Love for Arts and Sciences is the more conſpicuouſly ſeen, and you will appear upon the Liſt with thoſe worthy Patriots, whoſe Generoſity and Benevolence prompt them daily to ſupport and encourage all Sorts of Literature.

As I am ſenſible your *Honours* expect Nothing from me but Thanks and Duty, the firſt I moſt humbly return; and as I am conſcious I have hitherto been faithful and diligent in your Service, I hope I ſhall always endeavour, to the beſt of my Abilities, to continue ſo, in due Senſe of the many Kindneſſes I have received from you.

That the ſame kind Providence that recommended me to your *Honours* Favour, may continue to each Member the Bleſſing of Health, and that of Proſperity to the Company in general, is, without Doubt, the hearty Deſire of many; but of none more than,

Gentlemen, Your Honours moſt obliged,

and obedient humble Servant,

LONDON,
March 25, 1750.

D. FENNING.

PRE-

PREFACE.

To the SCHOOL-MASTERS of *Great-Britain*, and all Lovers of the Mathematical Learning.

GENTLEMEN,

MANY and great are the Advantages that ariſe from the Knowledge of common Arithmetic *only, to be Maſter of which requires a great deal of Time and Practice; but Algebra ſo far exceeds it, that were it not but that ſome ſenſible Progreſs muſt be made in the* Former, *it would hardly bear any Compariſon with the* Latter.

It may indeed be thought a needleſs and no leſs vain Attempt to offer any Thing of this Sort to the Public, *that has already been treated of by ſo many eminent Authors, as if more could be ſaid upon the Matter than has already been done. But it is to be obſerved, that the following Work is deſigned only as a Preparative to the right Underſtanding of other Authors; ſo that perhaps, upon due Conſideration, the Undertaking may be found to be neither impertinent nor improper: For the Queſtion is not,* Are there not Books enough extant? *but,* Whether the Generality of them are fit for Learners, ſo that they may be capable in a ſhort Time of themſelves to ſolve Queſtions without the Help of a Maſter? *This I think is the grand Point to be conſidered.*

I do not mean in the leaſt to leſſen thoſe Authors, for whoſe Works I would be thought to have the higheſt Regard; but from the little Progreſs that moſt make who undertake to learn Algebra *barely by Book, one would conclude (and that without the leaſt Detraction) that the Rules, Inſtructions, and Examples, are not ſo plainly ſuited to the Capacities of Learners as could be wiſhed.*

A great many are very ambitious to purchaſe for their very firſt Book ſome great and noted Author (and indeed ſuch muſt be conſulted, if we would become Maſters of any Art or Science;) but it is a common Miſtake to think they are beſt, and moſt to be depended upon in the Rudiments or fundamental Principles of ſuch Arts and Sciences. It is evident there is ſomething more in this than barely aſſerting it; for if it were ſo, what ſhould be the Reaſon that ſo many inferior Authors (perhaps of ten Times leſs Knowledge in the Art or Science itſelf) have wrote, and do ſtill con-

tinue to write Volumes, for the better underſtanding and explaining their Works?

If indeed there was no Occaſion for this, *then have they ſpent their Time in vain, and their Labours to no Purpoſe: But certainly they have not done ſo; it is what was wanting, and therefore a very neceſſary Undertaking; becauſe, by thus doing, the Unlearned have, and ſtill may, become acquainted with ſuch* Mathematical Books, *which otherwiſe they never would underſtand, without having a Tutor always at their Elbow; and then indeed* Pardie's Geometry *may be underſtood as well as* Leadbetter's Mathematical Companion, *or* Boad's Artium Principia: *And* Wolfius *and* De Billy, *as well as* Cocker *or* Hammond, *but not without* this, *or ſome ſuch like Advantages. Beſides,*

It has long ſince been allowed, that it is none of the eaſieſt Things for Men of profound Learning to write within the ordinary Compaſs of common Capacities: Their Knowledge will very rarely ſuffer them to ſtoop to the Underſtanding or Conception of ſuch: So that what Dryden *ſays of* Love *and* Wiſdom *together, may in another Senſe be applied to the Caſe before us:*

"The Proverb holds, that to be wiſe and love,
"Is hardly granted to the Gods above."

For indeed, to Men of ſuch extraordinary Parts and Abilities, every Thing of this Sort is ſo very eaſy, that whatever they propoſe, be it either Precept or Example, they can diſtinguiſh it ſo many different Ways, that from hence they conclude it is very eaſy to be underſtood by others; on which Account they are in general not only ſo ſublime, but withal ſo conciſe, that it is well known, that not only moſt of their eaſieſt Problems *(as they are pleaſed to call them) but even their Demonſtrations, require a further Demonſtration and more eaſy Explanation, before moſt Learners can of themſelves form a clear and diſtinct Idea of the Nature of the Propoſition; at leaſt it will coſt them many a weary Hour, and ſometimes Days and Months, to underſtand the whole Operation, though they apply ever ſo cloſe and aſſiduouſly; and all this for want of a few Words applied in a free, natural, and eaſy Manner. Indeed, when we conſider theſe Things, we cannot but ſay, it is a Pity a diligent Reader ſhould ſpend his Time to ſo little Purpoſe.*

As Algebra *is noted for its Excellency, ſo it is for its Difficulty, and therefore ſeveral eminent Authors have been pleaſed to call it a* dark *and* dry *Study; the Meaning I apprehend to be this, becauſe the Learner goes on a long Time through a Series of Rules and Examples, which, though he be ever ſo perfect in, yet ſees no Reaſon for what he has done, nor receives any Reliſh or Satisfaction, till he comes to put them in Practice in the ſolving of a few Queſtions; and even* then, *if the Author does not give a Reaſon for almoſt every Step of the Operation, the young* Tyro *will not ſo ſoon underſtand the Work as he may imagine, but is often at a Loſs, and ſometimes totally ſtopped.*

Now, if the young Algebraiſt *may poſſibly by ſome Authors proceed thus far, and for want of better Aſſiſtance can go no further, what can be thought of ſuch Books as have not ſufficient Inſtructions for the very Rudiments only. And that there are ſuch, is evidently known to too many, as will appear from the Words of an Author himſelf**.— “ *I have always been of Opinion, ſays he, that* Algebra “ *ſhould not be entangled with a great Number of Precepts;* “ *the Science is dark enough, without adding to it new* “ *Obſcurity by the Confuſion of different Operations,* &c.” *Had he ſaid* confuſed *Operations, indeed, it had been to the Purpoſe; but what he means by calling different Operations a Confuſion, I know not: For Reaſon (I think) tells us, that a Variety of Examples plainly demonſtrated, is the moſt ready Way to remove Obſcurity, as it is the only Means to prevent a Learner's being entangled; becauſe, without Examples, he would have but imperfect Ideas, and conſequently could never underſtand the* practical *Part of any Art or Science: It is therefore not only diſingenuous to own a Thing to be hard, and not withal to give ſufficient Rules and Examples to render it eaſy when in our Power; but it is as abſurd to think it ſhould ever be rightly underſtood without theſe.*

He is no great Arithmetician *who will not allow the Doctrine of* Fractions, *and the* Extraction *of* Roots *to be ſomething more difficult than the lower Rules of common* Arithmetic; *for which Reaſon the Directions are ſo many*

---

* *It is of no Signification to mention the Author's Name. Thoſe that have him by them, may ſee the ſame Words; and I make no Doubt but they will find he has kept up to them throughout his Work.*

the more, and ſo much the fuller: If then to underſtand theſe only, ſuch a Number of Caſes and Examples are neceſſarily required, it is reaſonable to ſuppoſe Algebra *is to be learnt without, or by a few only? No ſurely; for look but into the Works of the* unparalleled Saunderſon, *you will there find Precept upon Precept, and Example upon Example; and were it ſo that two Volumes in* Quarto *could be purchaſed by every Lover of this Science, they would (in all Probability) make more* Algebraiſts *than* all *the Books extant, would for ſome Years put a Stop to any further Pretences, and may be ſaid to be almoſt a* finiſhing *Stroke upon the Subject.—But this cannot be done.*

Now, contrary to this, ſeveral Authors are ſo over and above ſhort and intricate, that it is almoſt impoſſible for a Beginner to learn ſo much as the Algorithm, *much leſs the* Algoriſm.[*]

What may be done by One in a Thouſand is not eaſily accounted for. Heaven has favoured ſome with a ſurpriſing quick Apprehenſion, a penetrating Judgment, and tenacious Memory; and if to theſe we add the Advantages of Time, together with the Delight *and* Operoſity *that ſuch a one may poſſibly take in any Branch of Learning, ſuch a Man as* this, *I ſay, cannot fail to make ſenſible Improvements from the moſt ſublime Authors, and the moſt intricate Demonſtrations: But can we expect to find this at every Door? Very few have two of all theſe Advantages, and therefore* that *which is but* A, B, C, *to the* one, *is as* Greek *to the other.*

From a due Conſideration of theſe Things, it is eaſy to perceive that a Book wrote in a plain and familiar Manner, and with a moderate Price, has been long wiſhed for and expected; the Want of which (as ſeveral Schoolmaſters, and others my Acquaintance, have often ſaid) has been a very great Diſcouragement to Learners in their intended Deſign and Purſuit of Algebra.

The moſt rational Method to make any one Maſter of any Art *or* Science, *is certainly to introduce it to him in a natural Order, and to teach him at* firſt *only ſo much of it as is moſt neceſſary and conſiſtent with the Foundation; and when he underſtands the Fundamentals, he will ſoon be able to conquer the more difficult Parts, and may purſue the*

---

[*] Algorithm *ſignifies the firſt Principle, and* Algoriſm *the practical Part, or knowing how to put the* Algorithm *in Practice.*

Study in a more learned Manner, if his Fancy, Inclination, or Profeſſion, ſhall incline him ſo to do.

Such then is the Deſign of the following Sheets, to give the Learner a true Notion of Simple Equations; *and to make the Work as uſeful to him as I poſſibly could, I have added that Part which treats of* Vulgar Fractions, *leſt he ſhould not be acquainted with ſo neceſſary a Step: And tho' I have laid down every Thing in the firſt four* Algebraic *Rules as plain as I am able, ſtill, that the young Practitioner may not be at a Loſs in working the* Problems, *I have there recalled him back to his former Work, have given him the Reaſon* gradatim *(Step by Step) throughout the whole Operation, that he may the more readily underſtand the Nature of what he has been doing.*

I expect ſome critical Adepts *may ſay, there was no Occaſion to be ſo very particular; but let them be told once more, it is deſigned only for Learners, (though they themſelves, perhaps, may find ſome Things in it not altogether unworthy their Notice:) And I perſuade myſelf, upon this Conſideration, I have not dwelt too long upon any one Thing that requires a clear Demonſtration, if it were only for this Reaſon, that conſidering the many Rules and Examples the Learner has got to go through before he can put them in Practice, and the Difficulty of putting them in Practice to his own Satisfaction, an Author may be, and often is, too* ſhort *for moſt, but he never can be too* plain *for any; and I think the* practical *Part of Algebra has already been ſufficiently proved to be inconſiſtent with too much Sublimity and Conciſeneſs. Again,*

Does not Reaſon itſelf tell us, that Arts *and* Sciences *are not like* Hiſtory. *A few Words in the different Circumſtances and Parts of the* Narration *give us a general Idea of the Whole: But* here *it is quite otherwiſe; the Reader muſt have Words as it were continually multiplied, to underſtand truly what is before him; ſo that* that *which may be called a needleſs* Tautology *in the* one, *is Nothing more than a proper neceſſary* Repetition *or* Addition *to the* other.

I hope, therefore, thoſe more ſkilled in this Science *will excuſe my being a little prolix, if I tell them I have done it in* Sympathy *to the young Practitioner, ſince I know the dear Purchaſe of ſtudying* Algebra *from conciſe and obſcure Books by woful Experience!*

It is true, I have attempted the Work in Dialogues, which render it more prolix than it would have been without them; but as I had never ſeen any Thing of this Sort wrote in that Manner, I did it partly to avoid the Charge of Plagiariſm; *and it is the general Opinion, that this Way of Writing conveys the Senſe of the Matter ſooner to the* Ideas, *as it unbends the Mind at Intervals, not by turning it aſide from the Subject itſelf, but inſenſibly ſteals upon the Fancy, and renders the Study in a great Meaſure a Diverſion rather than a dry, burthenſome Taſk. But a benevolent* Critic *knows,*

> "Whoever thinks a faultleſs Piece to ſee,
> "Thinks what ne'er was, nor is, nor e'er ſhall be."

And a candid Judge, when he conſiders the Scope of the Science, and Deſign of the Author, will grant,

> That if the Way be juſt, the Conduct true,
> Some Praiſe, in Spite of trivial Faults, is due.

Let therefore the ingenuous Reader conſider, that every Day has its Shades, and that my chief End is to ſerve him, and to ſave him Trouble in the Purſuit of this excellent Study, and then he will for the very Deſign's Sake, forgive thoſe Errors which the Preſs, Want of Time, or Ability, may poſſibly have occaſioned.

*Gentlemen, permit me to return Thanks, to ſuch of you as have encouraged the three laſt Editions; and I have, (according to Promiſe) added to this Third Impreſſion, not only a plain and eaſy Method of compleating the Square, and extracting the Roots, but have alſo added ſeveral new Problems, by Way of Exerciſe, in order to make Quadratic Equations appear ſtill more eaſy to Learners.**

I am,
GENTLEMEN,
Your humble Servant,
and Well-Wiſher,

Royal Exchange,
London, Nov. 5, 1758.

D. FENNING.

---

* In this fifth Edition, the Appendix has been corrected in ſeveral Places; and Quadratic Equations and the Extraction of Roots are more fully treated of in the Supplement.

Juſt Publiſhed,

And Sold by S. Crowder, in Pater-noſter-Row, London,
And all other Bookſellers in Town and Country,
All written by the late ingenious Mr. D. FENNING,

A New Edition, being the Tenth, corrected and improved by W. Burbidge, Mathematician, Saliſbury, and recommended by ſeveral eminent Writing-Maſters and Accomptants, as the beſt Book of the Kind extant,

The Britiſh Youth's Inſtructor:

OR, A NEW AND EASY GUIDE to PRACTICAL ARITHMETIC.

Wherein the Rudiments of common Arithmetic, Vulgar and Decimal Fractions, the Extraction and Uſe of the Square and Cube Roots, &c. are ſo eaſily treated of, and ſo plainly demonſtrated, that any Perſon may of himſelf (in a ſhort time) become acquainted with every Thing neceſſary to the Knowledge of Buſineſs. Price 3s.

The Fourth Edition, greatly corrected and enlarged by Mr. Moon, Mathematician, Saliſbury,

The Young Man's Book of Knowledge;

Being a proper Supplement to the

YOUNG MAN's COMPANION.

In Six Parts, viz.

Part I. Of Knowledge in general.
II. Theology, containing an Account of the Religion and Laws of Nature.
III. Natural Philoſophy in general.
IV. Geography in a Manner entirely new; containing a general Deſcription of the four Quarters of the World.
V. Geometry and Aſtronomy, Navigation, and Plain Sailing.
VI. Of Muſic and Vibration.

Price 3s. neatly bound.

The Fifth Edition, with large Corrections and Improvements, particularly Six large and accurate Maps of the World and its different Parts, and alſo a Map of Great-Britain and Ireland,—recommended by ſeveral eminent Mathematicians.

A New and Eaſy Guide to the USE of the GLOBES, And the Rudiments of GEOGRAPHY.

Wherein the Knowledge of the Heavens and Earth is made eaſy to the meaneſt Capacity; firſt by giving a conciſe Account of the four Quarters of the World; and ſecondly, by the Solution of ſeventy uſeful Problems in Geography, Aſtronomy, Navigation, &c.
Price, neatly bound, 3s. 6d.

INTRODUCTION.

CHAP. I.

DIALOGUE I.

Between PHILOMATHES and TYRUNCULUS.

Tyr.——(at Philomathes's *Door*——*knocks.*)
Servant.——

Tyr. PRAY is *Philomathes* within?

Serv. Yes, Sir.

Tyr. Has he Company?

Serv. No, Sir: Pleaſe to walk into the Parlour, my Maſter is quite alone.

Tyr.——(*within*) I chooſe rather you would let him know I am here.

Serv.——(*to* Philomathes) Sir, here's a Gentleman deſires to ſpeak with you, if you be at Leiſure.

Phi. Who is it, *Pſapho?*

Serv. I have ſeen the Gentleman before, Sir, but I forget his Name.

Phi. —— (*comes.*)

Tyr. Dear *Philomathes*, I am your humble Servant.

Phi. *Tyrunculus*, I am yours, and am heartily glad to ſee you.—Come, Sir, pray walk this Way, I beg of you.

Tyr. — Sir —

Phi. Pleaſe to ſit here, Sir, for you ſeem to be cold

Tyr. Sir, I thank you; I am a little cold, I confeſs.

Phi. Give me Leave, *Tyrunculus*, to repeat once more, that I am proud to ſee you; and I hope, after the many Promiſes you have made, you are now come on purpoſe to ſpend an Hour or two with me.

Tyr. Indeed, *Philomathes*, I came with that Deſign, if it be convenient; if not, I beg I may wait upon you at a more ſuitable Opportunity.

Phi. Indeed, Sir, I ſhall conſent to no ſuch Thing; you ſhall not put me off from Time to Time with your Apologies; you ſee I am alone; why then ſhould you think it is inconvenient?

Tyr. Pardon me, *Philomathes*; as I ſee your Books before you, I thought you might be too buſy at this Time.

Phi. Piſh,—I am never ſo engaged with them, but that I am always ready to receive my Friend; though I confeſs (when I have not the Pleaſure of an agreeable Companion) they are pretty Company to me of themſelves.

Tyr. That I believe; for come when I will, you are always upon the Study.

Phi. Indeed I am no cloſe Student, *Tyrunculus*; 'tis true, ſome Things require more Thought and Conſideration than others, and I believe none more than what I have now been reading.

Tyr. Pray what is that, if I may be ſo free?

Phi. No Secrets at all, *Tyrunculus*; you may look in any of the Books and welcome.

Tyr. So, ſo! *Algebra!* —— *Cocker*, *Hammond*, *Saunderſon*, *Kerſey*, *Ward*, *De Billy*, *Wolfius*, &c. You have a Variety of Authors, upon my Word.

Phi. The Science requires it, *Tyrunculus.*

Tyr. I believe ſo, and a fine Science it is; for my Part, I wiſh I underſtood it; but it is ſo hard for a Learner to ſtep into it of himſelf, that I think it is quite tireſome.

Phi. You ſurprize me, *Tyrunculus*; why, you talked of learning long ago.

Tyr. I did ſo, and bought two or three good Pieces (as they are called) upon it; but I own I am very little the wiſer for all my ſtudying.

Phi. Do not ſay you ſtudied; I fancy you only gave them a careleſs Look or two; and Things of this Nature ſhould be read ſlow, and with the greateſt Attention.

Tyr. I am ſorry, *Philomathes*, you think I have taken no Pains; many an Hour's Reſt have I broke, to learn only the firſt four Rules; and as for Equations, I never could underſtand them; and to ſay the Truth, I have laid the Thoughts of it aſide for ſome Months.

Phi. I beg, *Tyrunculus*, you would not be angry; what made me think ſo is, becauſe I know moſt Learners are apt to run over Things too haſtily, and then blame the Author for their not coming at them directly: However, it is plain the Fault is either in you or the Books (if not in both) that you have made no better Improvement.

Tyr. I am apt to think the Fault is in them as much as in me.

Phi. Perhaps ſo; but what ſignifies barely aſſerting it, without giving a Reaſon?

Tyr. Whether I be expert enough to give you a ſufficient one, I will not ſay.

Phi. Let me hear you give ſome Reaſon or other, I beg.

Tyr. Why then, *Philomathes*, two of my Authors treat only of *Algebra* itſelf, beginning with *Problems* directly; the other indeed begins at *Addition*, and proceeds on to the firſt four *Algebraic* Rules, which I learnt pretty well; but when I came to *Algebraic Fractions*, I (not knowing any Thing of *Vulgar*, and he being ſo conciſe) could underſtand very little of them; on which Account, when I came to Equations, I was quite at a Loſs; for I perceive there are very few but what have Fractions, and I know very little of them, except juſt to read them.

Phi. Why indeed, *Tyrunculus*, if you are not acquainted with *Vulgar Fractions*, it is in vain you pretend to ſtudy *Algebra*, for they are the very Baſis and Foundation of it; however, *Algebraic Fractions* are done after the ſame Manner as *Vulgar*.

Tyr. But you will allow them to be much the harder of the two, I imagine?

Phi. Yes, yes, I grant it; but ſtill when the Learner is well acquainted with *Vulgar*, he will ſoon underſtand *Algebraic Fractions*; beſides, it will ſave him a great deal of Trouble, for it is impoſſible to reduce an Equation, in order to diſcover the Value of the unknown Quantity; without underſtanding one or both of theſe.

Tyr. Since you grant this then, *Philomathes*, I think you muſt own the Fault to be in the Books rather than in me?

Phi. 'Tis true, the Books you ſpeak of are not fit for Learners; but ſtill, I muſt not indulge you ſo much as to lay all the Fault upon them; for, as I ſaid before, though they may not be immediately fit for Beginners, yet they may be very good Books; for you are to conſider, *Tyrunculus*, that ſome Authors ſuppoſe Perſons previouſly to underſtand ſuch and ſuch Rules.

Tyr. You ſay very right, *Philomathes*; but pray did you never hear any beſides me complain of the Difficulty of learning *Algebra* by moſt Authors?

Phi. Yes, a great many, and there may be Reaſons for it; but we ſhould always, in ſuch Caſes, paſs our Sentiments with Reaſon, Caution and Tenderneſs, and not blame Authors on Account of every ſupine Learner; for it is evident we can have no better, if we go to the Extent of the Science; tho' I muſt confeſs thus far, that, upon the Rudiments, the Generality are a little dark, and too conciſe.

Tyr. Since you own this, *Philomathes*, how then could you ſo ſeverely blame me, that I have made no better Progreſs?

Phi. You are not ſo much to be blamed as I thought; but ſtill you are to be blamed in this Reſpect, that you have not conſulted more Books; for he that deſigns to be Maſter of any Art or Science, ſhould certainly provide himſelf with a Sufficiency for ſuch an Undertaking; for it often happens, that what one Author is deficient in, or treats darkly upon, another ſets forth in a clear and eaſy Manner to be underſtood.

Tyr. A Variety of Books cannot be had, you know, without Expence.

Phi. You make me ſmile to hear you talk of Expence; you know very well you can afford to purchaſe any Books you have a Mind to.

Tyr. That may be, but I ſpeak in Pity to thoſe that cannot; for by Reaſon of this, many Minds lie quite uncultivated, which otherwiſe would make fine Improvements in ſeveral Branches of Learning: This was the Caſe of *Tyro*, when he took in Hand to learn this Science, and I made him a Preſent of *Cocker* and *Hammond* to encourage him.

Phi. You did very kindly, and could not have made him a prettier Preſent; and he will certainly learn, for he has a very pretty Turn of Thought for Figures.

Tyr. I have heard *Novitius* ſay the ſame, (for they both practice together) but yet he ſays, that he never could rightly underſtand in what Manner *Cocker* reduces ſeveral of his Equations; for my Part, I wonder at his Patience.

Phi. Delight, *Tyrunculus*, Delight carries us through many Difficulties: But pray, do you remember any particular Queſtion or Equation that he ſeems ſo much puzzled about?

Tyr. That I do not: but I heard him wiſh he had the Happineſs of being acquainted with you, he would have aſked you how to reduce a certain Equation or two, which puzzles him pretty much, but that he feared you would take it amiſs.

Phi. Not I, in the leaſt: you know, *Tyrunculus*, I am of no ſuch ſelfiſh Temper; I hate it of all Things; it would be Ingratitude not to communicate that freely, which I received ſo: Baſe and ſordid Spirits will indeed deny their Aſſiſtance, that they may have the Pleaſure of laughing at the Ignorance of thoſe that they ought to have inſtructed.

Tyr. It is very true, *Philomathes*, *Philautus* is of this unhappy Diſpoſition; I heard *Novitius* ſay, a little while ago, he aſked him only a ſingle Queſtion, and he would not reſolve it, but gave him very little anſwer, and ſeemed, (he ſaid) to be affronted; and yet you know they are intimately acquainted, and he always expreſſes the greateſt Regard for *Novitius* in other Reſpects.

Phi. It is very ſurpriſing! The various Tempers of Men are not eaſily to be accounted for, you know, *Tyrunculus*: To be ſure, that Man can be of no Service to any Society, that is not ready and willing to aſſiſt every Member of it, and eſpecially when he is entreated. *Lucretius* was wont to ſay, "That he "would ſeek all Opportunities to communicate what-"ever he thought might be ſerviceable to any Man; "and that, if Wiſdom and Knowledge were given to "him with that *Reſerve*, that he might not impart it "to others, he ſaid, he would much rather chooſe to "be without them."

Tyr. ——— (*ſmiles*)

Phi. What do you ſmile at, *Tyrunculus*?

Tyr. Nothing, Sir.—— I was only going to ſay, I wiſh *Lucretius* lived near me.

Phi. That is not amiſs, *Tyrunculus*, I confeſs: However, I am as ready to ſerve you as he would be; and to ſhew you I am, if you approve of it, and have a Mind to have a little Touch at *Algebra*, I will give you the beſt Inſtructions I am capable of, upon Promiſe you will apply diligently; for were I ſure you would not, I ſhould repent my Folly even in aſking you, much more in the Undertaking itſelf. You remember the old Proverb, *Strike the Iron while it is hot*: If you ſlight this Offer, you may, perhaps, afterwards blame yourſelf.—— Come, what ſay you?

Tyr. Dear *Philomathes*, I am ſtill more obliged to you; and as I am bound in Duty to accept of your Kindneſs with Thanks, give me Leave to ſay, I will uſe my beſt Endeavours not to fruſtrate your kind Benevolence; but indeed it is giving you too much Trouble.

Phi. Do not mention it: You are welcome, as I ſaid before; but pray when will you begin, for the ſooner the better, if you take my Advice?

Tyr. That muſt be as you pleaſe, Sir.

Phi. It never can be more convenient than now, as we are alone, and free from any Interruption.

Tyr. With all my Heart, Sir.

Phi. Well then, *Tyrunculus*, I would have you obſerve the Method I ſhall take for your Inſtruction: I ſhall firſt begin with you at *Vulgar Fractions*, (as you have, you ſay, but very little Notion of them) and ſhall treat more of them than is required in the *Algebraic* Part, that you may ſee their Uſe in other Reſpects: Then I ſhall proceed to *Algebraic Fractions*, the Rule of *Proportion*, and *Equations*; wherein I ſhall give you ſeveral Examples very rarely to be met with, or ſo eaſily demonſtrated: After theſe, I ſhall make ſome neceſſary Obſervations, and proceed directly to *Algebraic Problems*; and ſhall work them ſo gradually, that you cannot miſs (if you take any Pains at all) to underſtand every Operation. But as ſeveral Things will, no Doubt, happen, that you may not immediately, upon firſt reading, have a true Notion of; pray do ſo much Juſtice to yourſelf, as to aſk the Meaning of every Thing you are at a Loſs for, and do not content yourſelf to go away half-taught. For my Part, I ſhall be careful to avoid any Thing that I think may give Occaſion to ſtop you in the Undertaking: Be you but as diligent to obſerve the Rules and Examples, and you will ſoon be Maſter of *that* which I ſhall hereafter inſtruct you in; and then, *Tyrunculus*, *Cocker* and *Hammond* will at once appear both beautiful and eaſy to you; or *Saunderſon* and *Kerſey*, if you think fit to purchaſe them, (the former of which I more particularly recommend:)* But if you have not the Happineſs to meet with either of theſe, you are here qualified for *Ward*, and have a good Foundation to peruſe more conciſe Authors, ſuch as *De Billy*, *Geſtinus*, *Wolfius*, &c. &c.

---

* As the Author makes no Mention of *Maclaurin*'s Treatiſe of Algebra, which is undoubtedly the beſt in the Engliſh Language, I take this Opportunity of recommending it. EDITOR.

DIALOGUE II.

SECT. I.

On NOTATION *and* REDUCTION *of* VULGAR FRACTIONS.

Tyr. WHAT do you mean by *Notation of Fractions?*

Phi. Notation ſhews you how to write down and expreſs any *Fraction.*

Tyr. What is a *Vulgar Fraction?*

Phi. A *Fraction* ſignifies a broken Number, or, in other Words, when Unity, or the Number 1, is divided into any Number of Parts, thoſe Parts are a *Fractional* Part of the Integer itſelf.

Tyr. How am I to know a *Vulgar Fraction?*

Phi. Whenever you ſee any Figure or Figures, with another Figure or Figures underneath, and a Daſh between them, (thus, $\frac{2}{3}$ $\frac{5}{4}$) they are *Vulgar Fractions* of ſome Denomination or other.

Tyr. What! are there different Sorts then?

Phi. Yes, three at leaſt.

Tyr. Tell me their different Names, if you pleaſe?

Phi. Firſt then, there are *Simple, Single,* or *Proper Fractions,* (for you muſt note they are frequently called by either of theſe Names;) *2dly, Improper;* and, *3dly, Compound Fractions.*

Tyr. How are they ſeparately known, or expreſſed in Figures?

Phi. Thus; $\frac{2}{3}$, $\frac{5}{8}$, $\frac{9}{14}$, $\frac{412}{716}$, are all *Proper Fractions;* they are ſo called becauſe each of the Numerators is leſs than the Denominator belonging to it.

Tyr. What do you mean by Numerator and Denominator?

Phi. The Numerator always ſtands a-top of the Daſh, and the Denominator underneath: Thus in the foregoing *Proper Fractions,* 2, 5, 9, and 412, are Numerators:

rators; and 3, 8, 14, and 716, are their reſpective Denominators.

Tyr. Very well: What is an *Improper Fraction?*

Phi. *Improper Fractions*, contrary to *Proper* ones, have their Numerators larger than the Denominators: Thus, $\frac{3}{2}$, $\frac{12}{7}$, $\frac{64}{9}$, and $\frac{716}{49}$, are *Improper Fractions.*

Tyr. What do you mean by *Compound Fractions?*

Phi. *Compound Fractions* are *Fractions* of *Fractions*, compounded, coupled, or joined together, by the Word *of*: Thus $\frac{2}{3}$ of $\frac{3}{4}$, or $\frac{5}{6}$ of $\frac{7}{11}$ of $\frac{21}{56}$, are *Compound Fractions*: Do you underſtand it?

Tyr. Yes, I do Sir: But how are theſe different *Fractions* read, or verbally expreſſed?

Phi. Thus, $\frac{2}{3}$ and $\frac{14}{29}$, is two Thirds, and fourteen Twenty-ninths; alſo $\frac{14}{5}$ is fourteen Fifths; and $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{11}$, is 3 Fourths of 5 Sixths of 7 Elevenths.

Tyr. The 3 Fourths of the 5 Sixths of the 7 Elevenths: But of what?

Phi. Why, the $\frac{3}{4}$ of the $\frac{5}{6}$ of the $\frac{7}{11}$ Parts of an Integer, or whole Number.

Tyr. I aſk Pardon: But it muſt be very hard to tell that.

Phi. You are not to concern yourſelf about this at preſent; you will find it eaſy enough by-and-by.

Tyr. Is there no other Sort of Fractions?

Phi. Properly ſpeaking there is not; but there is what we call a *Mixt Number.*

Tyr. What is that pray?

Phi. A *Mixt Number* conſiſts of two Parts, the firſt Part a whole Number, and the other a *Fraction*: Thus, $4\frac{2}{9}$, and $247\frac{5}{8}$, are *Mixt Numbers*; that is, 4 Units and $\frac{2}{9}$ of Unity or 1; and 247 Units and $\frac{5}{8}$ of 1.

Tyr. I underſtand it very well: What is the next Thing you purpoſe?

Phi. Nothing more concerning the Names of *Fractions*: I ſhall now give you three or four Obſervations, which you will do well to remember.

OBSERVATION I.

The Value of every *Proper Fraction* is leſs than Unity or an Integer, by ſo many Times as the Numerator is contained in the Denominator; as you will ſee demonſtrated (*Caſe* 9) in *Reduction:* So alſo is the Value of all *Compound Fractions* if they be compounded of *Proper* ones; for they are all but one *Simple Fraction* when reduced, as you will ſee (*Caſe* 6) in Reduction.

OBSERVATION 2.

Contrary to this, the Value of any *Improper Fraction* is more than an Integer, or as many whole Integers as the Denominator is contained Times in the Numerator; (See *Caſe* 9 in *Reduction.*)

OBSERVATION 3.

When the Numerator and Denominator are alike, this is called by ſome an *Improper Fraction*, but with what Propriety I know not, ſeeing it is only Unity itſelf: For $\frac{4}{4}$ of a £. Sterling is 1 £. and $\frac{9}{9}$ of a Yard 1 Yard; becauſe 4 divided by 4, and 9 divided by 9, gives one whole Integer.

OBSERVATION 4.

When you would make any whole Number into an *Improper Fraction*, then put Unity underneath it. Thus 4 will be $\frac{4}{1}$, and 126 will be $\frac{126}{1}$, &c. Pray remember this.

Tyr. I underſtand you quite well; but pray how is the Value of a *Fraction* diſcovered, in Order to know what Relation it bears to an Integer?

Phi. Fractions are reduced by certain Rules or Caſes in *Reduction*, of which if you be Maſter, you will ſoon *add*, *ſubtract*, *multiply*, and *divide*; but not elſe.

Tyr. Why is *Reduction* taught before the others pray?

Phi. Becauſe the *Fractions* muſt be firſt reduced before you can do the other Rules: *Reduction* therefore prepares the *Fractions*, as you will ſee by the following Examples.

SECT. II.

REDUCTION *of* VULGAR FRACTIONS.

Tyr. I AM mightily pleaſed with what you have ſhewn me concerning the Nature of *Vulgar Fractions*; but think I long to know how to reduce them.

Phi. That you ſhall directly; but are you ſure you know what a *Simple*, *Compound*, *Improper Fraction*, and *Mixt Numbers* are? for they muſt be known: And if you think you do not underſtand them quite well, give them another Look; you cannot be too perfect.

Tyr. I am poſitive I underſtand what they mean.

Phi. Very well: Pray hand me that Slate then; I will reduce them in their Order before your Face, and you may try at other Examples, which you may ſet yourſelf at your Leiſure.

CASE 1.

To reduce a Mixt Number *to an* Improper Fraction.

The Rule is,

Multiply the whole Number by the Denominator of the *Fraction* belonging to it, and take in the Numerator; then under this Product ſet the Denominator; ſo is this *Improper Fraction* equivalent to the *Mixt Number* given.

EXAMPLE 1.

Reduce $4\frac{2}{5}$ *to an* Improper Fraction.

$$\begin{array}{c} 4\frac{2}{5} \\ 5 \\ \hline 22 \\ \hline 5 \end{array} \textit{ Anſ}$$

EXAMPLE 2.

Reduce $51\frac{2}{11}$ *to an* Improper Fraction.

$$\begin{array}{r} 51\frac{2}{11} \\ 11 \\ \hline 563 \\ \hline 11 \end{array}\ \textit{Anſ.}$$

EXAMPLE 3.

Reduce $576\frac{14}{24}$ *to an* Improper Fraction.

$$\begin{array}{r} 576\frac{14}{24} \\ 24 \\ \hline 2308 \\ 1153 \\ \hline 13838 \\ \hline 24 \end{array}\ \textit{Anſ.}$$

Tyr. He that can do common Multiplication may do this.

Phi. True; and he that can do common Diviſion may do the next, it being only the Reverſe of the former Caſe, as you'll ſee by the ſame Examples.

CASE 2.

To reduce an Improper Fraction *to its* equivalent, whole, *or* mixt Number.

The Rule is,

Divide the Numerator by the Denominator, and if any Thing remains, ſet it over the Denominator, for a new Numerator, and it is done.

Ex-

EXAMPLE 1.

Reduce $\frac{22}{5}$ *to its* equivalent, whole, *or* mixt Number.

5) 22

$4\frac{2}{5}$ *Anſ. See Ex.* 1. *Caſe* 1.

EXAMPLE 2.

Reduce $\frac{563}{11}$ *to its* equivalent, whole, *or* mixt Number.

11) 563

$51\frac{2}{11}$ *Anſ.*

EXAMPLE 3.

Reduce $\frac{13838}{24}$ *to its* equivalent, whole, *or* mixt Number.

24)13838(576 $\frac{14}{24}$ *Anſ.*
120

183
168

158
144

14

Note, When there is no Remainder in the Diviſion, then will the whole Number be equivalent, or equal to the given *Improper Fraction*. As for Example: Suppoſe I would reduce $\frac{56}{7}$ to its Equivalent; I divide 56 by 7, and the Quotient is 8; ſo is 8 equal to $\frac{56}{7}$; ſo alſo $\frac{1558}{82}$ is equal to 19. This is eaſily ſeen by the next Caſe.

CASE 3.

To reduce any whole Number *to an* Improper Fraction.

The Rule is,

Multiply the whole Number by any Figure at Pleaſure, and under the Product ſet the ſame Figure you multiply'd by, and you have an *Improper Fraction* equal to the given whole Number.

EXAMPLE 1.

Reduce 14 *to an* Improper Fraction.

$$\begin{array}{ccc} 14 & 14 & 14 \\ \underline{5} & \underline{9} & \underline{12} \\ \frac{70}{5}\ \textit{Anſ.} & \frac{126}{9}\ \textit{Anſ.} & \frac{168}{12}\ \textit{Anſ.} \end{array}$$

Here you ſee I multiply the whole Number by 5, by 9, or by 12, or any other Figure, and the *Improper Fractions* are all equal to each other, and are alſo equal to 14. Therefore this is an unlimited Queſtion, to which an infinite Number of Anſwers may be given, and all right: but if the Queſtion be propoſed thus, it will be limited, and can then have only one Anſwer. As for Example,

EXAMPLE 2.

Reduce 14 *to an* Improper Fraction, *whoſe Denominator ſhall be* 15.

Here as I am to have 15 for its Denominator, I am obliged to multiply by 15, and no other Figure.

$$\begin{array}{r} 14 \\ \underline{15} \\ 70 \\ \underline{14} \\ \frac{210}{15} \end{array}\ \textit{Anſ. equal to } 14\textit{, as before.}$$

CASE 4.

To reduce a Fraction *to its loweſt Terms, equal in* Value *to the* Fraction *given.*

The Rule is,

Divide the Numerator and Denominator by any Figure that will divide them both without any Remainder, and continue ſo doing till you can divide them no lower; ſo will this laſt Quotient be the loweſt Terms equal to the original given *Fraction.*

EXAMPLE 1.

Reduce $\frac{144}{216}$ *to its loweſt Terms.*

Diviſors	2	3	4	3	
Num.	144	72	24	6	2
Denom.	216	108	36	9	3

Anſ. $\frac{2}{3}=\frac{144}{216}$

EXAMPLE 2.

Reduce $\frac{576}{960}$ *to its loweſt Terms.*

Diviſors	6	8	4	
Num.	576	96	12	3
Denom.	960	160	20	5

Anſ. $= \frac{576}{960}$

Do you underſtand the Work?

Tyr. I underſtand all very well, but the two Lines you make after the Anſwers I don't rightly apprehend.

Phi. What this Mark (=) do you mean?

Tyr. Yes.

What = ſignifies. *Phi.* It is the Sign of Equality, it ſignifies that $\frac{2}{3}$ is equal to $\frac{144}{216}$; and $\frac{3}{5}$ is equal to $\frac{576}{960}$. You will frequently ſee it uſed by and by.

Note, After the ſame Manner are *Algebraic Fractions* abbreviated. For ſuppoſe I were to abbreviate or reduce $\frac{abb}{bc}$ to its loweſt Terms; it is only taking away ſuch Letters or Quantities as are alike out of the Numerator and the Denominator, and the Work is done. Thus in the above *Algebraic Fraction* $\frac{abb}{bc}$, I find *b* both in the Numerator and Denominator, therefore by taking *b* from both, I have $\frac{ab}{c}$ in its loweſt Terms $= \frac{abb}{bc}$. But this you will ſee more of in Caſe 4 of *Algebraic Fractions.*

Tyr. Is there no other Method of reducing a *Fraction* to its loweſt Terms, becauſe it is difficult to find Figures that will divide ſome *Fractions.*

Phi. 'Tis true, for that Figure which will divide one, will not perhaps divide the other; but there is a Way to tell what Figure will do it at one Operation.

Tyr. That muſt be mighty pretty; pray let's ſee it?

Phi. You ſhall.

CASE 5.

Another Way to reduce a Fraction *to its loweſt Terms at one Work.* (Euc. 7, *Pr.* 1, 2, 3.)

Divide the Denominator of the *Fraction* by its Numerator; and if any Thing remains, divide your former Diviſor by it; and if any Thing yet remains, divide your laſt Diviſor by *that:* Thus proceed till Nothing remains; and then ſhall your laſt Diviſor be a *Common-Meaſurer*, that will infallibly divide both the Numerator and Denominator of the given *Fraction*, and reduce it into its loweſt Terms at one Work.

Tyr. Pray give me an Example, and explain it in Words.

Phi. I will.

EXAMPLE 1.

Reduce $\frac{147}{252}$ *to its loweſt Terms by a* Common-Meaſurer.

Firſt, I divide 252 by 147 the Numerator, and it goes once, and 105 remains; this 105 I make now a Diviſor, and the laſt Diviſor (*viz.* 147) a Dividend, and find it contains once 105, and 42 remains; by this 42 I divide the laſt Diviſor 105, and find 21 remains; and laſtly, by this 21 I divide the laſt Diviſor 42, and find Nothing remains: So is the laſt Diviſor 21 a *Common Meaſure*, that will reduce the given *Fraction* $\frac{147}{252}$ to its loweſt Terms at once; for dividing the Numerator 147 by 21, I have 7 for a new Numerator; and dividing 252 by 21, I have 12 for a new Denominator; and thus I find $\frac{7}{\text{[illegible]}}=\frac{147}{252}$.

EXAMPLE 2.

Reduce $\frac{574}{861}$ *to its loweſt Terms by a* Common-Meaſurer.

```
                      574) 861 (1
                           574
                           ———
  Com. Meaſurer 287) 574 (2
                     574
                     ———
                      0

               Num.
          287) 574 (2 New Num.
               574
               ———
                0

              Denom.
          287) 861 (3 New Denom.
               861
               ———
                0
```

Anſ. $\frac{2}{3}=\frac{574}{861}$

Tyr. I underſtand it well; but pray ſuppoſe a *Fraction* cannot be abbreviated by a *Common-Meaſurer*, by its proving an Unit at laſt?

Phi. Why then it is in the loweſt Terms already: Such a one is $\frac{176}{547}$.

Tyr. Very well. Pray are there no more Caſes in Reduction.

Phi. Yes, here follows,

CASE 6.

To reduce a Compound Fraction *to a* Simple-one *of the ſame Value.*

The Rule is,

Multiply all the Numerators one into another for a new Numerator; then multiply all the Denominators together for a New Denominator; ſo ſhall this new *Fraction* be equal to the *Compound Fraction* given.

Tyr. This is ſo eaſy, I think I can do it directly; pray try me?

Phi. No Doubt; for it is only common Multiplication.

EXAMPLE I.

Reduce $\frac{2}{3}$ *of* $\frac{5}{6}$ of $\frac{3}{4}$ *to a* Simple Fraction.

Tyr. I ſet down all the Numerators thus, 2
5
—
Then I multiply all the Denom. 3 10
6 3
— —
18 30 New N.
4
— —
New Denom. 72
—

Anſ. $\frac{30}{72}=\frac{2}{3}$ *of* $\frac{5}{6}$ *of* $\frac{3}{4}$.

What this × Croſs ſignifies.

Phi. It is very right, *Tyrunculus*; but there is a Character uſed for Multiplication which will mightily ſhorten the Work, and take up leſs Room; beſides it is frequently uſed in *Algebra*. This is it (×), and it ſignifies that all the Numbers between which it ſtands are to be multiplied together. Thus $4\times6\times2=48$. Pray remember it.

Tyr. I know your Meaning immediately. Thus, 4 multiplied by 8 is 4×8, that is 32. So $3\times2\times5=30$. Is it ſo or not?

Phi. You are very right; now I'll try you with another Sum.

EXAMPLE 2.

Reduce $\frac{5}{7}$ *of* $\frac{4}{5}$ *of* $\frac{3}{4}$ *of* $\frac{11}{12}$ *to a* Simple Fraction.

Tyr. I multiply all the Numerators together,

Thus $5 \times 4 \times 3 \times 11 = 660$ for a new Numerator.

And $7 \times 5 \times 4 \times 12 = 1680$ for a new Denominator.

So is $\frac{660}{1680} = \frac{5}{7}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{11}{12}$ *Anſ.*

Phi. It is very well done, *Tyrunculus*; now we will proceed to

CASE 7.

To reduce Fractions *of unequal, or different Denominators, to* Fractions *of the ſame Value, having but one common Denominator to all the Numerators.*

The Rule is,

Multiply all the Denominators together for a common Denominator; then take each Numerator, (beginning at the firſt) and multiply it into all the Denominators except its own Denominator; ſo ſhall theſe different Products be new Numerators to the common Denominator, equal to that *Fraction* whoſe Numerator you multiplied into the Denominators, which place over the common Denominator, and the Work is done. Do you think you could do this directly?

Tyr. No, this is not ſo eaſy as the laſt Caſe. Pray give me one Example at large, and then I'll try.

What N. N. and C. D. ſignify.

Phi. I will. Pray remember N. N. ſignify new Numerator, and C. D. common Denominator.

Ex-

EXAMPLE 1.

Reduce $\frac{2}{3}$, $\frac{3}{5}$, *and* $\frac{5}{8}$, *to* Fractions *of the ſame Value, having a common Denominator.*

$\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{8}$.

Firſt 3	then 2	and 3	and 5
5	5	3	5
15	10	9	25
8	8	8	3
C.D. 120	N.N. 80	N.N. 72	N.N. 75

Anſ. $\frac{80}{120}=\frac{2}{3}$, *for* 80 *is* $\frac{2}{3}$ *of* 120. *Alſo* $\frac{72}{120}=\frac{3}{5}$, *and* $\frac{75}{120}=\frac{5}{8}$.

Tyr. 'Tis ſo plainly done, that I think I can do another Example.

Phi. Poſſibly you may, as you take good Obſervation.

EXAMPLE 2.

Reduce $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, *and* $\frac{3}{4}$, *to* Fractions *having a common Denominator.*

Tyr. Firſt then, 5 × 6 × 8 × 4 = 960 for a common Denominator. Then 4 × 6 × 8 × 4 = 768 N. N. And 5 × 5 × 8 × 4 = 800 N. N. Again 7 × 6 × 5 × 4 = 840 N. N. And laſtly 3 × 8 × 6 × 5 = 720 N. N. Theſe new Numerators placed over the common Denominator, I find the Anſwer to be $\frac{768}{960}=\frac{4}{5}$, $\frac{800}{960}=\frac{5}{6}$, $\frac{840}{960}=\frac{7}{8}$, and $\frac{720}{960}=\frac{3}{4}$. Is it right?

Phi. You ſurprize me, to ſee you ſo apt; ſee what Care is! You have no Occaſion for more Examples in this Caſe. We will paſs on then to

CASE 8.

To reduce Fractions *of one Denomination to another.*

This conſiſts of two Parts, *aſcending* or *deſcending*. And firſt of *aſcending*.

When a *Fraction* is given to be brought from a leſs to a greater Denomination, then ſet down the *Fraction*, and make a compound one of it, according to the Denomination it is to be brought into; and this *compound Fraction* is made by conſidering how many of the leſs make one of the greater; then reduce this *compound* to a *ſimple Fraction*, and you will have a *Fraction* of another Denomination, equal in Value to the given *Fraction*.

Tyr. This is a hard Caſe; this is not underſtood by bare reading.

Phi. It is harder than ſome of the reſt; but an Example or two will make it plain.

EXAMPLE 1.

Reduce $\frac{3}{5}$ of *a Penny to the Fraction of a £. Sterling.*

Now obſerve, as 12 Pence make a Shilling, and 20 Shillings a Pound, I make a *compound Fraction* of $\frac{3}{5}$ thus.

$\frac{3}{5}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a £.

Now this reduced to a *ſimple Fraction, viz.* $3 \times 1 \times 1 = 3$ N. N. and $5 \times 12 \times 20 = 1200$ N. D. So is $\frac{3}{1200}$ of a £.* = $\frac{3}{5}$ of a Penny.

OR,

Otherwiſe make a *compound Fraction* of it at once; that is 240 Pence make a £. Sterling. Then it will be $\frac{3}{5}$ of $\frac{1}{240}$; this reduced to a *ſimple Fraction*, is $\frac{3}{1200}$ of a £. = $\frac{3}{5}$ of a Penny as above.

EXAMPLE 2.

Reduce $\frac{3}{4}$ *of a Farthing to the* Fraction *of a Guinea.*

This will be $\frac{3}{4}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{21}$ of a Guinea.

Now $3 \times 1 \times 1 \times 1 = 3$ N. N. and $4 \times 4 \times 12 \times 21 = 4032$ N. D. So is $\frac{3}{4032}$ of a Guinea = $\frac{3}{4}$ of a Farthing.

2. DESCENDING.

In *deſcending* you are to conſider, that the *Fraction* is to be brought from a greater to a leſs Denomination; therefore, as you multiplied the Denominator of the

---

* This Fraction, when reduced to its loweſt Terms, will be $\frac{1}{400}$ of a £; and the Fraction in the next Example will be $\frac{1}{1344}$ of a Guinea. Such Reductions will ſometimes ſave many Figures in an Operation, as ſhall be ſhown further on. EDIT.

given Fraction by the Parts contained in the Integer in Reduction *aſcending*, ſo now here you muſt multiply the Numerator of the *given Fraction* by the ſame Parts, and you have the Anſwer. Or, which is all the ſame, only invert the Parts contained in the Integer, (that is, turn them topſy-turvy) and make of them a *compound Fraction* as before, then reduce it to a *ſimple Fraction*, and it is done.

EXAMPLE 1.

Reduce $\frac{4}{5}$ *of a £. Sterling to the* Fraction *of a Penny.*

Here I conſider that a Shilling is $\frac{1}{20}$ of a £. and a Penny $\frac{1}{12}$ of a Shilling, therefore I multiply the Numerator 4 by 20 and by 12, and the Product is 960, which I place over the Denominator, thus $\frac{960}{5}$. So is $\frac{960}{5}$ of a Penny equal to $\frac{4}{5}$ of a £. or 16 Shillings.

Or, by inverting the Parts as above directed, it will be $\frac{4}{5}$ of $\frac{20}{1}$ of $\frac{12}{1}$ a *compound Fraction*, which reduced to a *ſimple one*, *viz.* 4 × 20 × 12 = 960 N. N. and 5 × 1 × 1 = 5 N. D. To prove this we will try *Example* 2. of Reduction *aſcending*.

EXAMPLE 2.

Reduce $\frac{3}{4032}$ *of a Guinea to the* Fraction *of a Farthing.*

Here $\frac{3}{4032}$ of $\frac{21}{1}$ of $\frac{12}{1}$ of $\frac{4}{1}$.
Now 3 × 21 × 12 × 4 = 3024 N. N.

And 4032 × 1 × 1 × 1 = 4032 N. D. This *Fraction* abbreviated * is = $\frac{3}{4}$ of a Farthing. So is $\frac{3}{4}$ of a Farthing = $\frac{3}{4032}$ of a Guinea, as in *Example* 2 of laſt Rule.

What do you ſay to this Caſe, *Tyrunculus?*

Tyr. I think I underſtand it pretty well; however, I will look it over again, and try at other Examples.

Phi. Do ſo. Now, *Tyrunculus*, we are come to the moſt uſeful and pleaſant Caſe of all, which is to find the true Value of any *Fraction*.

Tyr. That I ſhall like to know.

---

* That is, reduced to its loweſt Terms. See Page 16. EDIT.

Phi. Pray obſerve carefully the Rules and Examples, and I dare ſay you will work any of them directly after me.

CASE 9.

To find the Value of a Fraction *in Money, Weight, or Meaſure.*

The Rule is,

Multiply the Numerator by the Parts contained in the Integer to which it belongs, always obſerving to begin with that Part neareſt related to the Integer; then divide by the Denominator, and if any Thing remains, multiply it by the next greateſt Part neareſt related to the Integer, and divide again by the Denominator. Thus proceed till you can reduce it no lower for Want of Parts in the Integer, and the Work is done.

Tyr. I muſt beg one Example at large.

Phi. You ſhall, and you will need no more to underſtand the Caſe.

EXAMPLE I.

What is the Value of $\frac{3}{32}$ *of a £. Sterling?*

Firſt, in Order to find the Value of this *Fraction*, I conſider the next neareſt Part related to a £. and I find it to be Shillings. Now becauſe 20 Shillings make a £. I multiply the Numerator 3 by 20, and it is 60, which I divide by the Denominator 32, and have one in the Quotient, which is 1 Shilling, and 28 remains; this I call $\frac{28}{32}$ of a Shilling. Now as 12 Pence make a Shilling, multiply 28 the Numerator by 12, and it makes 336, which I divide alſo by the Denominator 32, and the Quotient is 10, which is 10 Pence, and 16 remains; this I call $\frac{16}{32}$ of a Penny: then as 4 Farthings make a Penny, I multiply the Numerator 16 by 4, and it is 64, which I divide again by the Denominator 32, and the Quotient is 2 Farthings. So that I find $\frac{3}{32}$ of a £. to be 1*s.* 10*d.* $\frac{1}{2}$. *See the Work.*

3
20
——
32) 60 (1*s*.
32
——
28
12
——
32) 336 (10*d*.
32
——
16
4
——
32) 64 (2 *qrs*.
64
——
0

Anſ. 1*s*. 10*d*. $\frac{1}{2}$.

EXAMPLE 2.

What is the $\frac{4}{25}$ *of a Moidore?*

4
27
——
25) 108 (4*s*.
100
——
8
12
——
25) 96 (3*d*.
75
——
21
4
——
25) 84 (3 *qrs*.
75
——
9

Anſ. 4*s*. 3*d* $\frac{3}{4}$ $\frac{9}{25}$.

Queſtions

Queſtions to be tried.

3. What's the Value of $\frac{4}{12}$ of an Cwt?	*Anſ.* 1 *qr.* 9 ℔. 5 *oz.* 5 *drs.* $\frac{4}{12}$.
4. What's the $\frac{127}{317}$ of a Year?	*Anſ.* 145 *d.* 5 *h.* 31 *m.* 36 *ſ.* $\frac{168}{317}$

☞ *To find the Value another Way.*

Suppoſe I wanted to know the $\frac{5}{12}$ of a £. or the $\frac{7}{18}$ of a Moidore.

This is done only by abbreviating the Numerator, and bringing it down to Unity, then will the *Fraction* be $\frac{1}{12}$ of a £. then multiply the Value by the Numerator, and you have its true Value.

Firſt, What is the $\frac{5}{12}$ of a £. Sterling?

I abbreviate the Numerator to Unity or 1, and conſidering what $\frac{1}{12}$ of a £. is, I find it to be 1*s.* 8*d.* this I multiply by 5 the Numerator, and it makes 8*s.* 4*d.* which is the Value of $\frac{5}{12}$ of a £. Sterling. Again,

What is the $\frac{7}{18}$ of a Moidore.

I conſider that $\frac{1}{18}$ of a Moidore is 1*s.* 6*d.* then by multiplying by the Numerator 7, I have 10*s.* 6*d.* which is the Value of $\frac{7}{18}$ of a Moidore.

N. B. I told you in *Dial.* the 2*d*, *Obſerv.* 1, that the Value of every *proper Fraction* was leſs than an Integer or Unity, and that the Value of an *improper Fraction* was more. (*Obſerv.* 2.) To prove which let us take any two *Fractions*, one *proper*, and the other *improper*, and ſee what their Value is in Relation to a £. Sterling.

EXAMPLE I.

Suppoſe the proper Fraction to be $\frac{9}{12}$ of a £.

I find $\frac{1}{12}$ of a £. to be 1*s.* 8*d.* therefore $\frac{9}{12}$ is 9 Times 1*s.* 8*d.* = 15*s.* Now 15*s.* wants 5*s.* of the whole Integer or 1 £. therefore the *proper Fraction* $\frac{9}{12}$ is leſs in Value than the whole Integer or Unity, by 5 Shillings. Now contrary to this, an *improper Fraction*'s Value is more than the Integer itſelf; and its Value is greater or leſs according to the Largeneſs of its Numerator.

EXAMPLE 2.

What is the $\frac{25}{4}$ of a £. Sterling?

This being an *improper Fraction*, I divide the Numerator 25 by the Denominator 4, and it gives 6 whole Integers, or 6 £. now as one remains over, it is $\frac{1}{4}$ of a £. which is 5*s*. So I find $\frac{25}{4}$ of a £. to be 6 £. 5*s*. Do you underſtand it, *Tyrunculus?*

Tyr. Had you only ſaid it I might have been at a Loſs; but you have demonſtrated it ſo plainly, that I muſt be quite dull of Apprehenſion not to ſee the Nature of it.

Phi. I am glad you underſtand me; and pray do you think you underſtand all the 9 Caſes in *Reduction* ſo as to work them now off Hand; for if you do not (at leaſt all but the 8th, that being not ſo much wanted) I freely tell you that you will be at a great Loſs; for the next four Rules depend wholly upon a true Knowledge of *Reduction*.

Tyr. You do well, *Philomathes*, to take ſuch Care of me, and I hope every Learner will take your Advice; but for my own Part, I can ſafely ſay I underſtand *Reduction* quite perfectly.

Phi. Well, if ſo, we will proceed directly to *Addition*.

DIA-

DIALOGUE III.

Of ADDITION, SUBTRACTION, MULTIPLICATION, *and* DIVISION *of* VULGAR FRACTIONS.

SECT. I.

Of ADDITION *of* VULGAR FRACTIONS.

Tyr. HOW is *Addition* of *Fractions* performed?

Phi. By this one general Rule, *viz.* all *compound Fractions* muſt firſt be reduced to *ſimple ones*, and all *Fractions* to a common Denominator, (by *Caſe* the *6th* and *7th* of *Reduction*;) then add all the Numerators together as in common *Addition*, and place their Sum over the common Denominator; and if it be an *improper Fraction*, reduce it to a mixt Number (by *Caſe* 2. *in Reduction*) and you have the Sum of all the *Fractions.*

EXAMPLE 1.

Add $\frac{2}{5}$, $\frac{1}{5}$, *and* $\frac{4}{5}$ *together.*

Here becauſe the *Fractions* have all one common Denominator, I only add the Numerators 2, 1, and 4 together, and their Sum is 7, which I place over the common Denominator 5, and the Sum is $\frac{7}{5}$, an *improper Fraction*, equal to $1\frac{2}{5}$ *Anſ.*

EXAMPLE 2.

Add $\frac{5}{21}$, $\frac{6}{21}$, $\frac{14}{21}$ *and* $\frac{19}{21}$ *together.*

Anſ. $\frac{44}{21} = 2\frac{2}{21}$.

Tyr. This is mighty eaſy; the Rule is ſo plain one cannot well miſs.

Phi. Now I will ſet you a Queſtion, *Tyrunculus*?

EXAMPLE 3.

Add $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, *and* $\frac{5}{6}$ *together.*

Tyr. I am afraid you have puzzled me; but ſtay a little, let me find it out myſelf—I ſee how it muſt be done, I muſt reduce the *Fractions* firſt to a common Denominator, and then add the Numerators together as you have done above. Muſt I not?

Phi. You have it, I perceive.

Tyr. Firſt then, to reduce the *Fractions* to a common Denominator.

$\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$.

I multiply $3 \times 2 \times 4 \times 6 = 144$ C. D. Then $2 \times 2 \times 4 \times 6 = 96$ N. N. Then $1 \times 3 \times 4 \times 6 = 72$ N. N. Again, $3 \times 2 \times 3 \times 6 = 108$. N. N. Laſtly, $5 \times 4 \times 2 \times 3 = 120$ N. N. So that I find the new Numerators are as follows:

N. Numerators.
96
C. D. 144 72
108 and
120

Their Sum 396 which placed over the common Denominator 144, ſtands thus $\frac{396}{144}$; this by *Caſe* the 2*d*, in *Reduction*, $= 2\frac{108}{144}$, that is $2\frac{3}{4}$, the Sum of $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$.

Phi. It is quite right, *Tyrunculus*, you ſee what it is to mind the Rules given for Inſtruction; you will do *that* already which will coſt a careleſs Reader ten Times the Trouble. Come, I'll try you with another.

Tyr. With all my Heart.

EXAMPLE 4.

Phi. *Add* $\frac{2}{4}$ *of* $\frac{4}{5}$ *of* $\frac{3}{4}$ *and* $\frac{4}{6}$ *of* $\frac{5}{8}$ *together.*

Tyr. Let me ſee: I muſt firſt (by *Caſe* 6. of *Reduction*) reduce the *compound* to *ſimple Fractions*, and then the *Fractions* to a common Denominator, and proceed as before. Firſt then I multiply all the Numerators together, *viz.* 2 × 4 × 3 = 24 N. N. Then 4 × 5 × 4 = 80 N. D. So is $\frac{24}{80} = \frac{2}{4}$ of $\frac{4}{5}$ of $\frac{3}{4}$. Then the other *compound Fraction*, *viz.* $\frac{4}{6}$ of $\frac{5}{8}$ reduced, is $= \frac{20}{48}$. So the two *ſimple Fractions* to be added are $\frac{24}{80}$ and $\frac{20}{48}$.* Theſe I reduce to a common Denominator, as in *Example* 3, and find them to be $\frac{1152}{3840}$ and $\frac{1600}{3840}$. Then have I nothing to do but add the Numerators together, and their Sum is 2752, which I place over the common Denominator thus, $\frac{2752}{3840}$, and this I apprehend to be the Sum of $\frac{2}{4}$ of $\frac{4}{5}$ of $\frac{3}{4}$ and $\frac{4}{6}$ of $\frac{5}{8}$. Is it ſo or not?

Phi. It is, and I am proud to ſee you ſo well grounded in the Rules of Reduction: However, I think I can poſe you the very next Queſtion. Will you try at it?

Tyr. To be ſure I will; for I imagine you will ſhew me if I cannot do it?

Phi. You need not doubt—Come then.

EXAMPLE 5.

Add 4£. $\frac{3}{4}$, 7£. $\frac{3}{5}$ *of* $\frac{2}{3}$ *and* $\frac{5}{12}$ *together.*

Tyr. ——— ——— ———

Phi. I thought I ſhould puzzle you; however, be not diſcouraged.

---

* The *Fraction* $\frac{24}{80}$, in its loweſt Terms, is $= \frac{3}{10}$; and $\frac{20}{48}$ is $= \frac{5}{12}$. Theſe two *ſimple Fractions*, $\frac{3}{10}$ and $\frac{5}{12}$, when they are reduced to a common Denominator, will become $\frac{36}{120}$ and $\frac{50}{120}$; and their Sum will be $\frac{86}{120}$; and if $\frac{86}{120}$ be reduced to its loweſt Terms, it will be $\frac{43}{60}$, which is the Sum of $\frac{2}{4}$ of $\frac{4}{5}$ of $\frac{3}{4}$, and $\frac{4}{6}$ of $\frac{5}{8}$. It may now be ſatisfactory to the Learner to ſee, that $\frac{43}{60}$ is of equal Value with $\frac{2752}{3840}$; and here he has no more to do than reduce $\frac{2752}{3840}$ to its loweſt Terms, by the common Meaſure 64, and it will become $\frac{43}{60}$.

The Learner ſhould further obſerve, that $\frac{2}{4}$ is $= \frac{1}{2}$; and $\frac{4}{6}$ is $= \frac{2}{3}$. The *Problem* will then be, *Add* $\frac{1}{2}$ *of* $\frac{4}{5}$ *of* $\frac{3}{4}$, *and* $\frac{2}{3}$ *of* $\frac{5}{8}$ *together.*

EDIT.

Tyr. There is nothing that puzzles me but the laſt *Fraction* $\frac{5}{12}$, for I am not certain whether it belongs to the *compound Fraction* that ſtands before it, or whether it be a ſeparate *Fraction* by itſelf.

Phi. It cannot belong to the *compound Fraction*, becauſe the Word *of* is not between them. There are two Ways to do this and ſuch like Queſtions; but the ſecond is the ſhorteſt and eaſieſt Method in my Opinion. I ſhall therefore only tell you the Way to work it, and leave you to try it by yourſelf at large.* Obſerve then,

METHOD 1.

Reduce the *compound Fraction* $\frac{3}{5}$ of $\frac{2}{3}$ to a *ſimple one*, which is $\frac{6}{15}$. Then will the Sum be thus, Add 4 £. $\frac{3}{4}$, 7 £. $\frac{6}{15}$ and $\frac{5}{12}$ together. Now by reducing the *mixt Numbers* 4 $\frac{3}{4}$ and 7 $\frac{6}{15}$ into *improper Fractions*, I have $\frac{19}{4}$ and $\frac{111}{15}$. Then may the Sum be read thus; add $\frac{19}{4}$, $\frac{111}{15}$ and $\frac{5}{12}$ together. Theſe *Fractions* I reduce to a common Denominator (as in *Example* 3.) and find them to be $\frac{3420}{720}$, $\frac{5328}{720}$ and $\frac{300}{720}$. Theſe Numerators added together, and divided by the common Denominator 720, give 12 in the Quotient, and 408 remaining. So is the Anſwer, 12 £. $\frac{408}{720}$ = 12 £. 11*s.* 4*d.*

METHOD 2.

The ſecond Way is certainly the beſt, becauſe you have no Buſineſs to meddle with the whole Numbers, but only with the *Fractions*, and then add their Sum to the whole Numbers afterwards.

---

* It is ſuppoſed, that the Learner by this Time knows how to reduce mixed Numbers to *improper Fractions*, and *compound Fractions* to *ſimple ones*, having had ſo many Examples of both Kinds.

The *Fractions* are $\frac{3}{4}$, $\frac{3}{5}$ of $\frac{2}{3}$ and $\frac{5}{12}$. Now $\frac{3}{5}$ of $\frac{2}{3} = \frac{6}{15}$.* Therefore add $\frac{3}{4}$, $\frac{6}{15}$ and $\frac{5}{12}$ together.

Theſe *Fractions* reduced to a common Denominator will be $\frac{540}{720}$, $\frac{288}{720}$ and $\frac{300}{720}$. Theſe Numerators added together, and divided by the common Denominator 720, give 1 whole Number in the Quotient, and 408 remains. So is the Sum of the *Fractions* only 1 $\frac{408}{720}$, which I add to the whole Numbers as follows, and have the ſame Anſwer as above.

Add 4£.
7
1 $\frac{408}{720}$ *

Anſ. 12 $\frac{408}{720}$ *as before* = 12£. 11*s.* 4*d.*

Tyr. This Way is the beſt, I ſee; becauſe if the whole Numbers conſiſt of many Places of Figures, then by the firſt Method in reducing them to *improper Fractions*, there will be a great many Figures, and a great deal of Work to reduce them to a common Denominator afterwards; whereas, if the whole Numbers be ever ſo large, it makes no Alteration in this ſecond Way of doing it.

Phi. Your Notion is right, *Tyrunculus*; and I think you will ſoon learn to *ſubtract*, you are ſo perfect in *Addition*.

Tyr. I will do the beſt I can.

Phi. Well, *Tyrunculus*, who can deſire any further? You have done well hitherto, and I hope will continue it. We will proceed then to

---

* $\frac{6}{15}$ is = $\frac{2}{5}$; and $\frac{408}{720} = \frac{51}{90}$.——EDIT.

SECT. II.

Of SUBTRACTION *of* VULGAR FRACTIONS.

Tyr. I HAVE heard that *Subtraction* is the hardeſt Rule in *Fractions*.

Phi. It is counted ſo by moſt Learners; however, he that underſtands *Reduction* well, may ſoon do it, as you will preſently find.

Tyr. Pray how is it performed ?

Phi. The very ſame as *Addition*, ſave only you are to ſubtract inſtead of adding, according to the Nature of the Queſtion; but the Rule is the ſame. For all *compound Fractions* muſt be reduced to *ſimple ones*, and then all to a common Denominator; after which only take the Numerator of the *Fraction* to be ſubtracted, out of the Numerator of the other *Fraction*, and you have the Difference or Anſwer as in common *Subtraction*.

	Ex. 1.	Ex. 2.	Ex. 3.
From	$\frac{19}{64}$	$\frac{14}{19}$	$\frac{136}{144}$
Take	$\frac{11}{64}$	$\frac{11}{19}$	$\frac{98}{144}$
Anſ.	$\frac{8}{64}$	$\frac{3}{19}$	$\frac{38}{144}$
Proof.	$\frac{19}{64}$	$\frac{14}{19}$	$\frac{136}{144}$

Here in theſe three Examples, becauſe the *Fractions* have a common Denominator, I only ſubtract the Numerators as in common *Subtraction*, and place the Difference over the common Denominator for an Anſwer. I prove the Work alſo as in common *Subtraction*; for I add the Numerator of the Difference to the Numerator of the leſs *Fraction*, which if the Work be right, will be equal to the Numerator of the greater *Fraction*.

Tyr. I think this is more diverting than *Addition*. Now let me aſk you a Queſtion or two if you pleaſe.

EXAMPLE 4.

From $\frac{13}{15}$ *take* $\frac{4}{11}$.

Phi. Firſt, I reduce the *Fractions* to a common Denominator, and find them $\frac{143}{165}$ and $\frac{60}{165}$, then I ſubtract the Numerator 60 from the Numerator 143, and there remains 83; which placed over the common Denominator 165, gives $\frac{83}{165}$ for the Difference.

EXAMPLE 5.

From $\frac{4}{6}$ *of* $\frac{7}{8}$ *take* $\frac{2}{5}$ *of* $\frac{1}{3}$.

That is, from $\frac{28}{48}$ * take $\frac{2}{15}$. Theſe reduce to a common Denominator, it will be, from $\frac{420}{720}$ take $\frac{96}{720}$. Now theſe are prepared for Work; therefore by ſubtracting 96 from 420, I have 324 remaining. So is $\frac{324}{720}$ the Difference between $\frac{4}{6}$ of $\frac{7}{8}$ and $\frac{2}{5}$ of $\frac{1}{3}$.

Tyr. There can be nothing eaſier than theſe Examples; he that underſtands *Addition* cannot miſs *Subtraction* indeed. But ſtill I have taken Notice of one Thing, which perhaps if I mention you will laugh at me.

Phi. Why ſhould you think ſo; *that* would be highly baſe in me, when I have before deſired you to aſk me any Thing that you are doubtful of; therefore pray let's hear it, it may perhaps be of more Service than you are aware of?

Tyr. It is this then: In all the foregoing Examples I perceive that the Numerator of the *Fraction* to be ſubtracted is leſs than the Numerator of the *Fraction* you ſubtract from, which makes all the Examples quite eaſy: But ſuppoſe the Numerator of the *Fraction* to be ſubtracted be larger than the other *Fraction*, where can I take it out of then, and how muſt I proceed in ſuch a Caſe?

Phi. You were afraid I ſhould laugh at you, but I aſſure you it is a very material Queſtion, for this is the moſt difficult Part of *Subtraction* that Learners meet with. The Rule then is this:

---

* $\frac{28}{48}$ is $= \frac{7}{12}$: and conſequently the reſt of the Work may be ſhortened. The Anſwer is $\frac{9}{20}$ which is $= \frac{324}{720}$.——EDIT.

N. B. When the Numerator of the lower *Fraction* (that is, the *Fraction* to be ſubtracted) is larger than the *Fraction* you ſubtract from, then take the ſaid Numerator out of the common Denominator, and to that Difference add the top, or leſs Numerator, ſo ſhall this be a new Numerator to be placed over the common Denominator, and you muſt carry one for borrowing out of the common Denominator, as you do when you borrow in common *Subtraction*, when the lower Figure is larger than the top one.

Tyr. This is quite plain, and eaſy enough to be performed, I ſhould think.

Phi. Eaſy; can a Thing be hard, when the Rule laid down to work it by tells you how to proceed in every Reſpect? However, I will try you with a Queſtion.

EXAMPLE 6.

From 24£. $\frac{5}{10}$, *take* 19£. $\frac{6}{10}$.

Tyr. I try to ſubtract the Numerator 6 from the Numerator 5, but cannot; therefore by the Rule I take 6 out of the common Denominator 10, and there remains 4, to which I add the leſs Numerator 5, and that makes 9, which I place over the common Denominator 10, and it is $\frac{9}{10}$. Then becauſe I borrowed out of the common Denominator 10, I carry 1 to the whole Number 19, and it makes 20, which I ſubtract from 24, and there remains 4. So is the Difference 4£. $\frac{9}{10}$ as under.

EXAMP. 7.

	£.		£.
From	24 $\frac{5}{10}$	From	419 $\frac{9}{46}$
Take	19 $\frac{6}{10}$	Take	147 $\frac{27}{46}$
Anſ.	4 $\frac{9}{10}$	*Anſ.*	271 $\frac{28}{46}$
Proved as in Ex. 1.	24 $\frac{5}{10}$	*Proof.*	419 $\frac{9}{46}$

Phi. *A Proof of* EXAMPLE 6. *by common* Subtraction.

£. £. s.
From 24 $\frac{5}{10}$, that is, 24. 10, by Caſe 9. of *Reduction.*
Take 19 $\frac{6}{10}$, that is, 19. 12

Anſ. 4 $\frac{9}{10}$, that is, 4. 18 *Difference.*

Proof 24 $\frac{5}{10}$, that is, 24. 10

Tyr. This is pretty to ſee the Proof of one Rule of Arithmetic by another.

Phi. You may prove any of the four Rules in *Fractions* by common Arithmetic as well as *this*; for they will come exactly alike if you proceed in a right Manner. Well, *Tyrunculus*, you ſeem to me to be qualified for *Multiplication*, but I have a Fancy to try you with one Queſtion more, which will make you Maſter of *Subtraction*.

EXAMPLE 8.

A *lent* B 240 £. $\frac{2}{5}$ *of* $\frac{3}{7}$ *of* $\frac{5}{9}$; *and* B *paid him* 176 £. $\frac{5}{7}$ *of* $\frac{11}{12}$; *what is ſtill due to* A?

Tyr. I proceed thus:

A 240 £. $\frac{2}{5}$ of $\frac{3}{7}$ of $\frac{5}{9}$; and *B* 176 £. $\frac{5}{7}$ of $\frac{11}{12}$. Now $\frac{2}{5}$ of $\frac{3}{7}$ of $\frac{5}{9} = \frac{30}{315}$; * and $\frac{5}{7}$ of $\frac{11}{12} = \frac{55}{84}$. Then $\frac{30}{315}$ and $\frac{55}{84}$, reduced to a common Denominator, will be $\frac{2520}{26460}$ and $\frac{17325}{26460}$. Thus are the *Fractions* prepared, and will ſtand thus: £.

A lent *B* 240 $\frac{2520}{26460}$
B paid him 176 $\frac{17325}{26460}$. Then by *Ex.* 6.

I find the balance 63 $\frac{11655}{26460}$; which (by *Caſe* 9. in *Reduction*) is 63 £. 8*s.* 9*d.* 2*q.* $\frac{2268}{2646}$.

Phi. I muſt needs ſay it is a Pleaſure to me to inſtruct you, *Tyrunculus.* I have but one Thing more to obſerve to you, and then we will go to *Multiplication.*

---

* $\frac{30}{315}$ is $= \frac{2}{21}$; and therefore the reſt of this Operation may be ſhortened.——EDIT.

Note. When you are to add or ſubtract the *Fraction* of a Farthing or a Penny from the *Fraction* of a £. or Guinea, &c. then (by *Caſe* the 8th, in *Reduction*) reduce the *Fraction* of the one, and make it equivalent to the *Fraction* of the other, and add or ſubtract as the Queſtion requires, you have the Anſwer. And thus much for Subtraction.

SECT. III.

MULTIPLICATION *of* VULGAR FRACTIONS.

Tyr. HOW is *Multiplication* of *Fractions* performed?

Phi. As in common Arithmetic, ſo alſo here are two Parts or Factors given, *viz.* the *Multiplicand* and *Multiplier*. When therefore you have reduced the mixt Numbers to *improper Fractions*, and the *compound* to *ſimple Fractions*, the Rule is, multiply the two Numerators together for a *new Numerator*, and the Denominators together for a *new Denominator*, and you have the Product or Anſwer; which, if it be an *improper Fraction*, reduce to a mixt Number, and the Work is done.

EXAMPLE 1.

Multiply $\frac{4}{5}$ *by* $\frac{5}{8}$.

Anſ. $\frac{20}{40} = \frac{1}{2}$.

EXAMPLE 2.

Multiply $\frac{42}{56}$ *by* $\frac{24}{38}$.

Here $42 \times 24 = 1008$ N. N. and $56 \times 38 = 2128$ N. D. *Anſ.* $\frac{1008}{2128}$.

Tyr. Nothing is eaſier than this indeed. Pray try me with a few Queſtions?

Phi. I will; and I am fully perſuaded that you will work moſt of them, if you rightly obſerve the Rule, which pray look at once more, leſt you be not perfect in it.

EXAMPLE 3.

Multiply $\frac{4765}{9}$ *by* $\frac{14}{29}$.

Tyr. Firſt 4765 × 14 = 66710 for N. N. and 9 × 29 = 261 N. D. *Anſ.* $\frac{66710}{261} = 255\,\frac{155}{261}$.

Phi. You are very right.

EXAMPLE 4.

Multiply $\frac{5}{4}$ *of* $\frac{8}{5}$ *by* $\frac{3}{6}$ *of* $\frac{5}{9}$.

Tyr. Firſt $\frac{5}{4}$ of $\frac{8}{5} = \frac{40}{20}$; and $\frac{3}{6}$ of $\frac{5}{9} = \frac{15}{54}$; therefore I multiply $\frac{40}{20}$ by $\frac{15}{54}$, and the *Anſwer* is $\frac{600}{1080}$. Now I'll aſk you one, if you pleaſe.

Phi. Pray do.

EXAMPLE 5.

Tyr. *Multiply* 41 £. $\frac{4}{5}$ *by* 12 £. $\frac{3}{4}$.

Phi. Theſe being mixt Numbers, they are equal to *improper Fractions*, that is $41\,\frac{4}{5} = \frac{209}{5}$; and $12\,\frac{3}{4} = \frac{51}{4}$; that is, $\frac{209}{5}$ and $\frac{51}{4}$. Now 209 × 51 = 10659; and 5 × 4 = 20; ſo that the *Anſwer* is $\frac{10659}{20}$, which reduced to a mixt Number, is $532\,\frac{19}{20}$; that is 532 £. 19*s*.

Tyr. Now I think I can do any Queſtion in this Rule.

Phi. Perhaps ſo; but it runs in my Head that I can puzzle you in one of the two next Examples.

EXAMPLE 6.

Multiply 14 £. $\frac{3}{4}$ *of* $\frac{5}{6}$, *by* 9 £. $\frac{3}{5}$ *of* $\frac{15}{18}$.

Tyr. I know I can do this. Firſt, $\frac{3}{4}$ of $\frac{5}{6} = \frac{15}{24}$, *viz.* $= \frac{5}{8}$; and $\frac{3}{5}$ of $\frac{15}{18} = \frac{45}{90}$, *viz.* $= \frac{1}{2}$. Therefore I multiply 14 £. $\frac{5}{8}$ by 9 £. $\frac{1}{2}$. Theſe reduced to *improper Fractions* will ſtand thus; multiply $\frac{117}{8}$ by $\frac{19}{2}$. Now 117 × 19 = 2223 N. N. and 8 × 2 = 16 N. D. that is $\frac{2223}{16}$ *Anſ.* $= 138\,\frac{15}{16} =$ 138 £. 18*s.* 9*d.*

Phi. Very well done; and the better becauſe you abbreviated the *Fraction* $\frac{45}{90}$ and $\frac{15}{24}$; for which you will ſee the Reaſon given in the Method of *Abbreviations*, *Dialogue* 4. *Sect.* 2. Now for the ſecond Queſtion, *Tyrunculus.*

EXAMPLE 7.

Multiply 14 £. $\frac{5}{9}$ *by* 13 £.

Tyr. Let me ſee 14 £. $\frac{5}{9}$ by 13 £.—Why here is but one *Fraction given.*—I believe you have ſet me now indeed.

Phi. It was but this Inſtant that you ſaid, you believed you could do any Sum in the Rule.—Why don't you make a *Fraction* of the whole Number 13?

Tyr. I muſt know how firſt.

Phi. O for Shame, don't you know, ſo plainly as I told you in *Dial.* 2. *Obſerv.* 4. Pray don't forget your former Inſtructions, and then blame me; I thought this would put you to a Nonplus: However it is the firſt great Error you have committed, ſo we will paſs it by; but pray be more careful for the future. I told you, to make a *Fraction* of a whole Number, is only putting Unity under it. Thus 13 is $= \frac{13}{1}$.

Tyr. I bluſh to think I ſhould be ſo remiſs. However, pardon me, I know now eaſily how to perform it: For $14\frac{5}{9} = \frac{131}{9}$; and $13 = \frac{13}{1}$; therefore by multiplying $\frac{131}{9}$ by $\frac{13}{1}$, I have $\frac{1703}{9} = 189\frac{2}{9}$; that is, 189 £. 4*s.* 5*d.* 1*q.* $\frac{3}{9}$ or $\frac{1}{3}$ of a Farthing *Anſ.*

Phi. Very right, *Tyrunculus*; and now I ſhall make ſome Obſervations, which though already known to ſuch as are well verſed in *Vulgar Fractions*, yet, as they are not taken Notice of by any Author I am acquainted with, it may be ſerviceable to you and others.

NOTE I.

When one *proper Fraction* is multiplied by another, the Product will be a *proper Fraction*; therefore the Anſwer is leſs than Unity. It will be alſo leſs than Unity when one *compound Fraction* is multiplied by another, provided they be compounded of *proper Fractions.* (See *Obſerv.* 1. *Dial.* 2.) Thus in *Example* 1. where $\frac{4}{5}$ is multiplied by $\frac{5}{8}$, the Anſwer is but $\frac{1}{2}$; that is, $\frac{4}{5}$ of a £. multiplied by $\frac{5}{8}$ of a £. is but $\frac{1}{2}$ of a £. or 10 Shillings.

NOTE

NOTE 2.

Contrary to this, one *improper Fraction* multiplied by another is more than Unity. Thus if you take the *Fractions* in *Example* 1. and invert them, it will be $\frac{5}{4}$ of a £. multiplied by $\frac{8}{5}$ of a £. and the Anſwer will be $\frac{40}{20} = 2$ Integers, or 2 £. So that the Product now is juſt four Times more than it was before.

NOTE 3.

When a *proper Fraction* is to be multiplied by or with an *improper Fraction*, the Product will be ſometimes a *proper*, and ſometimes an *improper Fraction*. Thus $\frac{4}{9}$ multiplied by $\frac{11}{8}$ produces the *proper Fraction* $\frac{44}{72}$; but $\frac{9}{7}$ multiplied by $\frac{5}{6}$ gives $\frac{45}{42}$, an *improper Fraction*, for the Anſwer.

NOTE 4.

When the Numerator of one *Fraction* is equal to the Denominator of the other, and the Denominator of it equal to the Numerator of the other, then will their Product be Unity or 1. Thus $\frac{4}{9}$ muliplied by $\frac{9}{4}$, the Product is $\frac{36}{36} = 1$. (See *Dial.* 2. *Obſerv.* 3.) So alſo $\frac{7}{11}$ of a £. into $\frac{11}{7}$ of a £. $= \frac{77}{77} = 1$ £.

Tyr. Theſe Obſervations are of great Service to ground any one in the right Notion of *Multiplication.* But pray is there no Method to prove this Rule?

Phi. Certainly there is; and this is the Beauty of *Arithmetic*, that it admits of the Proof of itſelf divers Ways by divers Rules. For Inſtance, Suppoſe $\frac{28}{7}$ of a £. were multiplied by $\frac{15}{3}$ of a £. the Anſwer is $\frac{420}{21} =$ 20 £.

PROOF.

Now $\frac{28}{7} = 4$ £. and $\frac{15}{3} = 5$, and $4 \times 5 = 20$ £. as above, &c. &c. &c.

Tyr. I am highly obliged to you, *Philomathes*, for your Care in giving me ſo many Examples. Pray is there any Thing more worthy my Notice, or neceſſary to be known in *Multiplication?*

Phi. Nothing: I have indeed been more particular already than I intended; therefore I ſhall paſs directly to *Diviſion.*

SECT. IV.

Of DIVISION *of* VULGAR FRACTIONS.

Tyr. HOW do you divide one *Fraction* by another?

RULE 1.

Phi. After having reduced all *mixt Numbers* and *compound Fractions* as before directed, to *ſimple* or *improper Fractions*, the Rule is, Multiply the Numerator of the *Fraction* to be divided into the Denominator of the *Fraction* you divide by, and place their Product for a new Numerator; then multiply the Denominator of the Dividend into the Numerator of the Diviſor for a new Denominator, which place under the new Numerator for an Anſwer. *Or,*

RULE 2.

If you invert the Diviſor, that is, turn it into contrary Order, by ſetting the Numerator underneath, and the Denominator over it; them multiply the Numerators and Denominators together, as in *Multiplication*, and you have the ſame Anſwer as above.

EXAMPLE 1. *by* RULE 1.

Divide $\frac{4}{9}$ *by* $\frac{7}{11}$. *Anſ.* $\frac{44}{63}$.

Same EXAMPLE *by* RULE 2.

Multiply $\frac{4}{9}$ *by* $\frac{11}{7}$. *Anſ.* $\frac{44}{63}$.

EXAMPLE 2. *by* RULE 1.

Divide $\frac{14}{9}$ *by* $\frac{6}{11}$. *Anſ.* $\frac{154}{54} = 2\frac{46}{54}$.

Same EXAMPLE *by* RULE 2.

Multiply $\frac{14}{9}$ *by* $\frac{11}{6}$. *Anſ.* $\frac{154}{54}$ *as before.*

Tyr. I like the ſecond as well as the firſt Way.

Phi. Uſe which you pleaſe, provided you are but perfect in either. There is no Occaſion for any more Examples, ſeeing that you have a Rule both for *ſimple* and *compound Fractions.* However, I'll give you an Example or two more by Way of Exerciſe.

EXAMPLE 3.

Divide 41 £. $\frac{3}{5}$ *by* 6 £. $\frac{5}{8}$; that is, *divide* $\frac{208}{5}$ *by* $\frac{53}{8}$.

Anſ. $\frac{1664}{265} = 6$ £. $\frac{74}{265}$; that is 6 £. 5*s.* 7*d.* $\frac{5}{265}$. And after the ſame Manner for *compound Fractions.*

Tyr. I underſtand you quite well. Pray is there any Thing elſe to be obſerved in *Diviſion?*

Phi. I ſhall make a few Remarks upon the Rule itſelf, which may be of Service.

NOTE 1.

When an *improper Fraction*, having Unity for its Denominator, is to be divided by a *proper Fraction*, whoſe Numerator is alſo Unity, the Quotient will always be an *improper Fraction*, having Unity for its Denominator, and therefore conſequently equal to a *whole Number.*

NOTE 2.

When the Denominators or Numerators are not Unity, the Quotient will ſometimes be an *improper*, and ſometimes a *proper Fraction.*

NOTE 3.

From hence it is eaſy to perceive, that *Diviſion* of *Fractions* will anſwer the ſame End as common *Multiplication:* That is, a leſs Number may be brought into greater by this *Diviſion*, contrary to common *Diviſion*, *viz.* Moidores, Guineas, or Pounds Sterling into Pence and Farthings; or Hundred-Weight into Pounds and Ounces, *&c.*

Tyr. What do you ſay, Pounds may be brought into Pence and Farthings by *Diviſion?* I thought *Diviſion* had made any Number *leſs* and not *more!*

Phi. It is true, it does ſo in common *Diviſion*, but it is quite contrary in *Vulgar Fractions*; for here, *more* is brought into *leſs* by *Multiplication*, and *leſs* into *more* by *Diviſion*.

Tyr. I ſhould be glad to ſee an Example of this Sort if you pleaſe; for I have heard ſome great Pretenders to *Arithmetic* ſay it cannot be done, 'tis contrary to Reaſon.

Phi. Pleaſe then to propoſe a Queſtion yourſelf?

EXAMPLE 4.

Tyr. *It is required to bring* 30 *Moidores into Farthings by* Diviſion *only?*

Phi. And cannot you do it, think you?

Tyr. ——Why really at preſent I am at a Loſs.

Phi. Pray be pleaſed to read over *Note* 1. in *Multiplication* and *Diviſion*, for it is only for want of being perfect in them, and truly underſtanding the Nature of different *Fractions.*

Obſerve then,

As one Part of the given Queſtion is Farthings, and the other Moidores, I reduce (by *Caſe* 8. in *Reduction*) a Farthing to the *Fraction* of a Moidore, which *Fraction* I make a Diviſor; and the whole Number 30 I make a *Fraction* of alſo for a Dividend; ſo will the Quotient be the true Anſwer. *See the Work.*

By *Caſe* 6. of *Reduction* $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{27} = \frac{1}{1296}$ of a Moidore for a Diviſor; and 30 will be $\frac{30}{1}$. Now $\frac{30}{1} \div \frac{1}{1296} = \frac{38880}{1}$; that is, 38880 Farthings. Do you underſtand it?

Tyr. Yes, quite well; I could not have thought it had been ſo eaſy: But pray what does this Mark $\div$ ſignify?

Phi.

What $\div$ *ſignifies.* *Phi.* It is the Sign of *Diviſion*, and ſhews, that the Number before it is to be divided by the Number after it.

Tyr. Very well. Pray try me with a Queſtion of this Sort?

EXAMPLE 5.

Phi. *It is required to bring* 84 *Guineas into Farthings by* Diviſion *only?*

Tyr. I find (by *Caſe* 6. of *Reduction*) a Farthing to be equal to $\frac{1}{1008}$ of a Guinea. Then $\frac{84}{1} \div \frac{1}{1008} =$ 84672 Farthings *Anſ.*

Phi. Very right, and pray remember, that the ſame is to be done by *Decimal Fractions*, by finding the Decimal of a Farthing at a Moidore or Guinea the Integer, and dividing the *whole Number* thereby you will have the ſame Anſwer.

From hence will naturally ariſe the ſelf-evident Truth of

NOTE 4.

That when any *whole Number* is divided by a *proper Fraction*, the Quotient will be ſo much larger than the Dividend as the Diviſor is leſs than Unity; but when a *proper Fraction* is divided by a *whole Number*, the Quotient will be ſo many Times leſs than the Dividend as the Diviſor exceeds Unity; thus 5 divided by $\frac{1}{4}$ is equal to 20; but $\frac{1}{4} \div 5 =$ but $\frac{1}{20}$.

And thus, *Tyrunculus*, having finiſhed theſe four Rules with Variety of Examples, I ſhall now exerciſe you in them with ſome practical Queſtions in the *Rule of Proportion*; which, if duly obſerved, will make you a complete Maſter of *Vulgar Fractions*.

DIA-

DIALOGUE IV.

SECT. I.

The RULE *of* THREE, *in* VULGAR FRACTIONS.

Tyr. I AM proud to think I am got thus far, and yet I almoſt dread the Queſtions you are going to ſet me.

Phi. If you be perfect in the foregoing Rules, you have no *Reaſon* to fear *this* at all; for it is nothing elſe but putting the others in Practice.

Tyr. Is not the Rule of Three of *Fractions* wrought in the ſame Manner as the common Rule of Three Direct.

Phi. The very ſame, due Regard being had to the *Fractions.* There are two Methods, the ſecond of which is (in general) the readieſt and eaſieſt; but you may take your Choice.

RULE 1.

Having reduced all *compound* to *ſimple Fractions*, and all *mixt Numbers* to *improper Fractions*, then ſtate your Queſtion by making the firſt and third Number of one Name or Denomination; this done, *Multiply your ſecond Number by your third, and divide by your firſt*, and you have the Anſwer. *Or,*

RULE 2.

Having reduced the *Fractions*, and placed the Numbers in Order as before directed, *Multiply the Denominator of your firſt Number into the Numerators of the ſecond and third for a new Numerator; then multiply the Numerator of the firſt Fraction or Number into the Denominator*

nominator of the ſecond and third, for a new Denominator, which place under the new Numerator for an Anſwer.

EXAMPLE I.

If $\frac{3}{4}$ *of a Yard coſt* $\frac{5}{6}$ *of a* £. *what coſt* $25\frac{5}{8}$ *Yards.*

$$\text{If } \tfrac{3}{4} \text{ ——— } \tfrac{5}{6} \text{ ——— } \begin{array}{c} 25\tfrac{5}{8}. \\ 8 \\ \hline 205 \\ \hline 8 \\ \hline \end{array}$$

Now $\frac{205}{8} \times \frac{5}{6} = \frac{1025}{48}$; this $\div \frac{3}{4} = \frac{4100}{144} = 28$£. $\frac{68}{144} =$ 28£. 9*s*. 5*d*. 1*q*. $\frac{48}{144} = \frac{1}{3}$.

Or, by RULE 2.

Having ſtated the Queſtion thus, If $\frac{3}{4} - \frac{5}{6} - \frac{205}{8}$. *I multiply the Denominator of my firſt Number (viz.* 4.*) into the Numerators of the ſecond and third (viz* 5. *and* 205.*) and it gives* 4100 *for a N. Numerator. Then I multiply the Numerator of the firſt (viz.* 3.*) into the Denominators of the ſecond and third, (viz.* 6. *and* 8.*) and it gives* 144 *for a N. Denominator. So is the Anſwer* $\frac{4100}{144}$, $=$ 28£. $\frac{68}{144}$ *as above.*

Tyr. I think as you ſay this ſecond Method is the beſt, if it were only becauſe it ſaves the Trouble of *Diviſion*.

Phi. It is the beſt Way in your plain eaſy Queſtions, but in ſome Reſpects the firſt Method is moſt practicable; however, either Way you ſee anſwers the ſame End, and therefore you may take either of them, as Practice or Fancy may direct.

Tyr. I ſhall take Care to be perfect in both. Pleaſe to try me with a Queſtion?

Phi. I will.

Ex-

EXAMPLE 2.

If a Load of Wheat coſt 7 £. $\frac{14}{24}$, *what coſt one Buſhel?*

Tyr. A Load being 40 Buſhels, I ſtate it thus:

If $\frac{40}{1}$ —— £. 7 $\frac{14}{24}$ —— $\frac{1}{1}$

$$\begin{array}{r} 24 \\ \hline 182 \\ \hline \end{array}$$

24 Now $\frac{182}{24} \times \frac{1}{1} = \frac{182}{24}$; this $\div \frac{40}{1} = \frac{182}{960}$ *of a* £. which reduced, the Value of it is 3*s.* 9*d.* $\frac{1}{2}$ *Anſ.*

SECOND WAY.

Firſt I reduce the *mixt Number* 7 $\frac{14}{24}$ to an *improper Fraction*, and it is $\frac{182}{24}$, as above, then the Numbers will ſtand thus:

If $\frac{40}{1}$ —— $\frac{182}{24}$ —— $\frac{1}{1}$.

Now 1 × 182 × 1 = 182 N. N. and 40 × 24 × 1 = 960 N. D. So is the *Anſwer* $\frac{182}{960}$ = 3*s.* 9*d.* $\frac{1}{2}$, as before.

Phi. Very well done, *Tyrunculus.*

EXAMPLE 3.

What is the Intereſt of 219 £. $\frac{3}{5}$ *for a Year at* 5 £. $\frac{3}{8}$ *per Cent?*

Tyr. —— Here I muſt crave your Aſſiſtance.

Phi. You ſhall have it in Words at Length.

Firſt, If $\frac{100}{1}$ ——— 5 $\frac{3}{8}$ ——— 219 $\frac{3}{5}$. My 2*d* and 3*d* Terms being *mixt Numbers*, I reduce them to *improper Fractions*, and they are $\frac{43}{8}$ and $\frac{1098}{5}$. Now $\frac{43}{8} \times \frac{1098}{5} = \frac{47214}{40}$; this ÷ the 1*ſt* Number $\frac{100}{1}$ gives $\frac{47214}{4000}$, which reduced to a *mixt Number* is 11 £ $\frac{3214}{4000}$, *viz.* 11 £. 16*s.* 0*d.* $\frac{3}{4}$ $\frac{1440}{4000} = \frac{144}{400} = \frac{9}{25}$. Try it at Leiſure by the ſecond Method.

Tyr. I could not have thought *Vuglar Fractions* had been ſo uſeful.

Phi. Nothing more neceſſary than theſe and *Decimal Fractions*, for the ready finding the Intereſt or Value of any Thing, eſpecially when the Queſtions are not in whole Numbers, as you will ſee by the following Examples.

EXAMPLE 4.

A Merchant makes an Aſſurance upon a Ship and Cargo (bound to a certain Part) valued at 4500 £. 15*s. and agrees to pay* 16 *Guineas* per cent. *what comes the Premium or Charge of the Aſſurance to?*

Firſt, 16 Guineas being 16 £. 16*s.* this in *Fractions* is 16 $\frac{16}{20}$ = 16 $\frac{4}{5}$; and 4500£. 15*s.* is 4500$\frac{3}{4}$. This and 16 $\frac{4}{5}$ reduced to *improper Fractions* will be $\frac{84}{5}$ and $\frac{18003}{4}$. Then will the Number ſtand thus:

$$\text{If } \frac{100}{1} \text{ —— } \frac{84}{5} \text{ —— } \frac{18003}{4}.$$

Now by *Rule* 2. 1 × 84 × 18003 = 1512252 N. N. And 100 × 5 × 4 = 2000 N. D. So is $\frac{1512252}{2000}$ the Anſwer, which reduced to a *mixt Number*, you have £. 756 $\frac{252}{2000}$ = 756 £. 2*s.* 6*d.* $\frac{480}{2000}$ or $\frac{48}{200}$.

EXAMPLE 5.

A *buys of* B £. 420 $\frac{2}{3}$ *Stock, and gives* £. 95 $\frac{4}{5}$ per Cent. *what comes it to?*

Firſt, 420 $\frac{2}{3}$ = $\frac{1262}{3}$ and 95 $\frac{4}{5}$ = $\frac{479}{5}$. Then,

$$\text{If } \frac{100}{1} \text{ —— } \frac{1262}{3} \text{ —— } \frac{479}{5}.$$

Anſ. $\frac{604498}{1500}$ = 402 £. 19*s.* 11*d.* 2*qrs.* $\frac{108}{150}$.

The Proof of this is worthy your Obſervation, *Tyrunculus*, to ſhew you the Beauty of *Fractions*, which I inſert purely for your Satisfaction, to give you a juſt Idea of Things of this Nature.

PROVED *another* WAY.

Firſt, £. 95 $\frac{4}{5}$ *per Cent.* wants £. 4 $\frac{1}{5}$ *per Cent.* of being *Cent. per Cent.* therefore the Queſtion may be read thus:

What comes £. $420\frac{2}{3}$ *to, deducting* £. $4\frac{1}{5}$ per Cent?

Then,

If $\frac{100}{1}$ —— $4\frac{1}{5}$ —— $420\frac{2}{3}$.

Work as in the laſt, and you will have $\frac{26502}{1500}$ = 17£. 13*s*. 4*d*. 1*q*. $\frac{420}{1500}$ *Anſwer*. Now this added to the foregoing Sum 402£. 19*s*. 11*d*. 2*qrs*. $\frac{108}{150}$ is equal to the original Stock propoſed, *viz*. £. $420\frac{2}{3}$ = 420£. 13*s*. 4*d*.

Tyr. Indeed, *Philomathes*, this is delightful; pray continue your Examples.

Phi. I am as ready to do it as you are to aſk; but remember, *Tyrunculus*, by Way of Digreſſion, *We muſt cut our Garment according to the Cloth:* I have already given more Examples than I intended; but ſtill, if you have any particular Queſtion to aſk me, I am ready to do any Thing that may be of Service.

Tyr. Sir, you are extremely kind to indulge me thus far; but what I have to add is this, That by what I have ſeen of this Rule, it muſt be very ſerviceable to tell the Nature or Proportion of Coins: Is it not?

Phi. To be ſure it is, eſpecially when the different Sorts of Exchange with ſome other current Money is not altogether equal to our Pound *Sterling*.

Tyr. I will aſk you a Queſtion then.

EXAMPLE 6.

A Merchant in Holland *draws a Bill upon his Correſpondent in* London *for* 4280 *Ducatoons, at* 6*s*. 3*d*. $\frac{3}{5}$ *each; what muſt he receive in Pounds* Sterling?

If $\frac{1}{1}$ —— 6*s*. 3*d*. $\frac{3}{5}$ —— —— $\frac{4280}{1}$.

Firſt, bring the ſecond Number into Pence, then multiply them by the Denominator 5, and take in the Numerator 3, ſo will the Numbers be $\frac{1}{1}$ —— $\frac{378}{5}$ and $\frac{4280}{1}$. Now, by *Rule* 2. 1 × 378 × 4280 = 1617840 N. N. And 1 × 5 × 1 = 5 N. D. So is $\frac{1617840}{5}$ the Anſwer is Pence, *viz*. 323568 = 1348£. 4*s*. 0*d*.

Tyr. Pray prove the Work by whole Numbers?

Phi. That is done very eaſily by *Practice*, or ſeveral other Ways. But for common Underſtanding, I know none better than this: Firſt, find the Value of 4280 Pieces, at 6*s.* 3*d.* (that is, at 75*d.*) each, and it is 321000*d.* Then you have to find the Value of 4280 Pieces, at $\frac{3}{5}$ of a Penny each; therefore by multiplying 4280 by 3, and dividing by 5, you have 2568 Pence, which added to the other, will give you the Anſwer as follows:

4280 Pieces at 75 *d.* = 321000
Ditto, at $\frac{3}{5}$ *d.* = 2568

Ditto, at 75$\frac{3}{5}$*d.* = 323568 *d.* = 1348 £. 4*s.* 0*d.*
as above.

Tyr. Then I am always to multiply by the Numerator and divide by the Denominator in ſuch Caſes; am I not?

Phi. I know no eaſier or ſhorter Way, I aſſure you.

Tyr. It is eaſy enough indeed, as you ſay, and I am obliged to you for ſo plain a Demonſtration: Give me Leave to aſk you a Queſtion ſtarted the other Day in my Company, and I'll have done. It is this:

EXAMPLE 7.*

A poor Man dying leaves 20 *Shillings to his four Sons,* A, B, C, *and* D; *to* A *he left* $\frac{1}{3}$, *to* B $\frac{1}{4}$, *to* C $\frac{1}{5}$, *and to* D $\frac{1}{6}$, *with a particular Charge that the whole might faithfully be diſtributed among them; it is demanded what each Legacy amounts to?*

Phi. I have not room to inſert the whole Work, but will plainly tell you the Method of doing any Thing of this Sort.

Firſt, I take the $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, of 20*s.* and adding the Parts together, I find the Sum but 19*s.* So that the *Executor* has 1*s.* in Hand, and the Will is not fulfilled. Then ſay, if 19*s.* give $\frac{1}{3}$, (*viz.* 6*s.* 8*d.*) what will 20*s.*

---

* This and the foregoing Examples were added by Deſire of a Friend.

give? Proceed thus with the reſt, and you will find the Anſwer to be as under:

A's Share	o	7	$0\frac{4}{19}$
B's ——	o	5	$3\frac{3}{19}$
C's ——	o	4	$2\frac{10}{19}$
D's ——	o	3	$6\frac{2}{19}$
Sum	1	o	$0\frac{0}{0}$ *

Had he left the $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and the $\frac{1}{5}$ of 20*s.* it would amount to 25*s.* 8*d.* which you ſee is 5*s.* 8*d.* more than 20*s.* Their Shares then of the 5*s.* 8*d.* are found as above, only now ſaying, If 25*s.* 8*d.* be 20*s.* what will $\frac{1}{2}$ or 10*s.* be, &c.

Note, By the ſame Method is found the particular Shares of every Creditor when a Man breaks or comes a Bankrupt; as alſo how much in the £. his Effects amount to.

Tyr. I return you hearty Thanks, Sir, for your Care, and do aſſure you that I underſtand the Nature of what I have ſeen very well.

Phi. I am glad of it, *Tyrunculus*; but you will be more confirmed in the Knowledge of *Fractions*, and will know better how to apply them to Uſe, if I ſhew you ſomething of the Nature of *Abbreviations*.

---

* Queſtions of this Sort, as they are commonly expreſſed, are but little better than Nonſenſe. For the Thing propoſed to be done, is not to find the $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, of 20s. but to divide 20s. into four Parts in Proportion to the Fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$. And if theſe be reduced to a common Denominator, in the loweſt Terms, they will become $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$, and $\frac{10}{60}$; and the Problem will be, To divide 20s. into four Parts, in Proportion to the Numbers 20, 15, 12, and 10; which may be eaſily done as below.

$$\begin{array}{c}\text{As the ſum of the Numbers}\\ 57 : 20\text{s.} ::\end{array}\left\{\begin{array}{l} 20 : 400 \div 57 = 7\ 0\ \frac{12}{57}\text{ A's ſhare.} \\ 15 : 300 \div 57 = 5\ 3\ \frac{9}{57}\text{ B's.} \\ 12 : 240 \div 57 = 4\ 2\ \frac{30}{57}\text{ C's.} \\ 10 : 200 \div 57 = 3\ 6\ \frac{6}{57}\text{ D's.} \end{array}\right.$$

Sum 20 0

EDIT.

SECT. II.

Of ABBREVIATIONS.

Tyr. WHAT is the Uſe of *Abbreviations?*

Phi. The right underſtanding of *Abbreviations* will mightily help to ſhorten the Work, and ſave a Multiplicity of Figures; beſides, without them it would be quite difficult to anſwer ſome Queſtions, at leaſt very tedious, as you'll ſee by and by.

Tyr. What, would you have me firſt abbreviate the *Fractions* before I begin the Queſtion?

Phi. To be ſure I would when they will allow of it; for if you remember you did it yourſelf in *Example* 6, in *Multiplication.*

Tyr. I believe I did; however, let me have an Example or two, that I may the better underſtand it.

Phi. You ſhall.

EXAMPLE 1.

If $\frac{48}{140}$ *of a Load Coſt* $\frac{70}{175}$ *of a* £. *what coſt* 12$\frac{36}{96}$?

Firſt, $\frac{84}{140} = \frac{3}{5}$, $\frac{70}{175} = \frac{2}{5}$, and $\frac{36}{96} = \frac{3}{8}$. Therefore the Sum may be thus read:

If $\frac{3}{5}$ *of a Load coſt* $\frac{2}{5}$ *of a* £. *what coſt* 12$\frac{3}{8}$ *Loads?*

Anſ. £. 8$\frac{3}{12}$ = 8£. 5*s.* Now is it not eaſier to work with $\frac{3}{5}$, $\frac{2}{5}$, and $\frac{3}{8}$, than with the original *Fractions* $\frac{84}{140}$, $\frac{70}{175}$, and $\frac{36}{96}$?

Tyr. I ſee the Thing, it appears quite plain to me now; and I ſhall take Care to obſerve it. Is this all you have to ſay upon *Abbreviations?*

Phi. No; I have ſomething more to add, wherein you will ſee the Beauty and Uſe of *Abbreviations* more plainly: Beſides, your Friend *Diſcipulus* deſired me to communicate it to you, as very few or none have taken Notice of it in their Writings.

Tyr. Pray what is it?

Phi. It is, *To know whether a* Fraction, *when abbreviated, (or reduced to its loweſt Terms) be equivalent in all Reſpects to the Original, or given* Fraction.

Tyr. I know no Method of proving it, but by comparing the Value of one with the Value of the other; thus I find $\frac{24}{36}$ of a £. is equal to 13*s.* 4*d.* and I find $\frac{2}{3}$ of a £. to be the ſame; therefore I conclude that $\frac{2}{3}$ is $= \frac{24}{36}$.

Phi. It is right; but ſuppoſe you could neither abbreviate a *Fraction*, nor yet find its Value, how then would you act?

Tyr. That I do not know.

Phi. Indeed, *Tyrunculus*, though you can work the Rule of Proportion pretty well, you are no great Judge of the Nature of it, I find. Obſerve then,

Firſt, *As the Numerator of the* Fraction, *in its loweſt Terms, is to its Denominator, ſo will the Numerator of the original or given* Fraction *be to its own Denominator: Or, as one Numerator to the other, ſo will one Denominator be to the other.*

EXAMPLE 2.

To prove whether $\frac{3}{5}$ *be equal to* $\frac{84}{140}$ *or* $\frac{127}{180} = \frac{30480}{43200}$.

Firſt, as 3 to 5, ſo is 84 to 140: Or, as 3 to 84, ſo is 5 to 140. Again, As 127 to 180, ſo is 30480 to 43200.

Tyr. I am yet more obliged to you, *Philomathes*; for this muſt infallibly prove what you have ſaid, ſure enough.

Phi. Since you are ſenſible of this, *Tyrunculus*, I will ſhew you another Way to prove it much ſhorter and eaſier than the former.

Second, *When you have reduced any* Fraction *to its loweſt Terms to prove whether it be right, multiply the Numerator of the original* Fraction, *by the Denominator of the abbreviated one; and the Denominator of the original, by the Numerator of the abbreviated one; and if the Products are equal, your Work is rightly performed.* Thus, take the two *Fractions* above, *viz.* $\frac{84}{140} = \frac{3}{5}$. For $3 \times 140 = 420$, and $5 \times 84 = 420$. Again $\frac{30480}{43200} = \frac{127}{180}$: For, 30480×180, and 43200×127 are both equal to 5486400.

Tyr. This is ſhort and eaſy indeed!

Phi. I will now give you an Example or two to ſhew you that the Knowledge of *Abbreviations* is of more Uſe than you imagined.

EXAM-

EXAMPLE 3.

A *and* B *are two Merchants, but of different Places*; A *owes* B 46£. 12*s.* 9*d.* *Now* 100£. *of* A's *current Money is equal to* 140£. *of* B's; *what muſt* A *pay to remit the aforeſaid Debt?*

See the Work.

£. *s.* *d.*
46 – 12 — 9
5

7)233 – 3 — 9

33 — 6 — 3

Tyr. This is ſhort indeed!

Phi. But you know the Reaſon (I hope) of my multiplying and dividing by theſe Figures; do you not?

Tyr. Stay, let me conſider a little upon it.——I ſee the Reaſon now plain enough. They are in Proportion to each other as the current Money of the Places are, I perceive; that is, $\frac{100}{140} = \frac{5}{7}$; for as 5 to 7, ſo is 100 to 140. Is not this the Reaſon?

Phi. Moſt certainly: For I can multiply and divide better by 5 and 7, than by 100 and 140; beſides, how much ſhorter is this, than to work it at Length by the *Rule of Three* direct. So alſo, if 108£. of *A*'s be equal to 45£. of *B*'s, then I multiply by 5, and divide by 12, becauſe $\frac{45}{108} = \frac{5}{12}$, and the Anſwer will be 19£. 8*s.* 7*d.* $\frac{3}{4}$. Try you at it, ſuppoſing 105£. of *A*'s be equal to 165£. of *B*'s.

Tyr. I will, Sir, and I heartily thank you for this additional and uſeful Obſervation.

Phi. The Pains I have taken I ſhall count a Pleaſure, if you make but a good Improvement: And let me perſuade you not to meddle with *Algebra* till you are perfect in *Fractions*; for if you do, you will not be able to make a right Judgment of the *Problems*, much leſs know how to do them. If you think you underſtand what you have done, there remains but one Thing more before you enter upon *Algebra*, and that is, that you learn the *Signs* and *Characters* therein uſed.

Tyr. You have ſhewn me ſome already.

Phi. I know it: But there are a great many more; therefore I ſhall write them down, and give you them Home, that you may learn them by-Heart before I ſee you again; which, though I am always glad of, yet, upon this Occaſion, I neither deſire nor expect, till you have learnt them ſo perfectly, as to know their Meaning the Moment you ſee them.

Tyr. You may depend upon it, *Philomathes*, in a ſhort Time: Where be they?

Phi. Stay a little.———Here, *Tyrunculus*, here they are, and I wiſh you well to learn them.

Tyr. I heartily thank you, *Philomathes*, and am your Servant.

Phi. I am yours, *Tyrunculus*.

SECT. III.

An EXPLANATION *of the* PRINCIPAL SIGNS *and* CHARACTERS *uſed in* ALGEBRA.

1. THIS Character (+) is the Sign of *Addition*, and ſignifies, that the Numbers or Quantities between which it is placed, are to be added together in one Sum. Thus, 3 + 5 ſhews, that 3 and 5 are to be added together. It ſtands for the Word *more* alſo. Thus, 5 + 4 + 7, is read, 5 more 4, more 7, which makes 16. So alſo, $a + b + c + d$, ſhews, that a, b, c, and d, are to be added together.

2. This Character (—) is the Sign of *Subtraction*, and ſignifies, that the Numbers or Quantities which come after it, are to be taken from the Numbers or Quantities which ſtand before it. Thus, $a + b - c$, ſhews, that the Quantity c is to be taken from the Sum of a and b. It ſtands for the Word *leſs* alſo. Thus, 9 — 5, is read, 9 *leſs* 5, which is 4; and $b - 9$ is b *leſs* 9, and ſhews that 9 is to be taken from the Quantity b.

Note farther, That (+) ſignifies a *poſitive* or *affirmative* Quantity, or *abſolute* Number; but (—) ſignifies a *fictious* or *negative* Quantity or Number; a Want or Deficiency. Thus — 8 is 8 Times leſs than Nothing. So that any Number or Quantity with the Sign + being added to the ſame Number or Quantity with the Sign —, their Sum will be equal to Nothing. Thus 8 added to − 8 is equal to (0) but − 8 taken from + 8 is = 16. (*See Caſe 2d. in Addition and Subtraction of Algebra.*) Again,

An Aſteriſk (*) is frequently uſed for a Cypher in *Subtraction*, that is, $b + c$ taken from $b + c$, there remains * or Nothing.

2. This Character (×) is the Sign of *Multiplication*. It ſignifies *into*, or *multiplied by*. Thus $4 \times 5 \times 3$, ſhews, that 4 is to be multiplied by or into 5, and their Product into or by 3. So alſo $a \times b \times c \times d$, ſhews the continual *Multiplication* of a, b, c, and d.

⁂ When Quantities are placed one after another, without any Sign or Character, it ſhews their *Multiplication*. Thus ab is $a \times b$, or a multiplied by b. So $abcd$ ſhews the Product of a into b into c into d. Joining of Quantities therefore is multiplying them together.

4. This Character (÷) is the Sign of *Diviſion*, and ſignifies that the Numbers or Quantities before it, are to be divided by the Numbers or Quantities after it. Thus $a \div b$, ſhews that a is to be divided by b; ſo $16 \div 4$, ſhews that 16 muſt be divided by 4.

Note. There is a better Way of expreſſing *Diviſion*, and it is more frequently uſed; and this is by placing the Dividend a-top, and the Diviſor underneath it. Thus a divided by b is ſet thus $\frac{a}{b}$. So alſo 16 divided by 4 is thus placed, $\frac{16}{4}$.

5. Theſe two Lines (=) are the Sign of Equality, and ſignify, that the Quantities and Numbers on the one Side of it are equal to the Numbers or Quantities on the other.

6. This Character (∺) is the Sign of *continued* or *Geometrical Proportion*. Thus, $a ∺ b ∺ c$, &c. are Quantities in *Geometrical Proportion*. But *Geometrical Proportion* is better expreſſed by the Powers of one and the ſame Quantity. Thus, a, aa, aaa, $aaaa$, a^5, a^6, &c. are Quantities in *Geometrical Proportion*. So alſo 2, 4, 8, 16, 32, &c. are in *continual Proportion*.

7. This Character (:) ſignifies the Word *to*; and this (::) ſignifies the Words *ſo is*. When they are joined together thus, (:: ::) they are the Rule of Proportion; and being placed between Numbers or Quantities, (thus, $a : b :: d : e$) they are thus read or expreſs'd, As *a* to *b*, *ſo is d* to *e*. Or thus, 4 : 6 :: 8 : 12,) is As 4 to 6, *ſo is* 8 to 12.

8. This Character (φ) is here uſed to ſignify *Tranſpoſition*, and ſhews that the Number or Quantity before which it is placed is in the next Line or Step tranſpoſed to the other Side of the Equation, with a contrary Sign.

9. This Character or Letter (Q) is alſo here uſed, and ſignifies *by the Queſtion*, as you may ſee in the Work of the following Problems.

10. This Character ($\surd$) is a radical Sign, or Sign of the Square Root, and ſhews that the Number or Quantity before which it ſtands is to have its Square Root extracted.

11. This Character ($^3\surd$) is the Sign of the Cube Root, and ſignifies the Extraction of it, as in the Square Root above.

SECT. IV.

Of COMPOUND SIGNS *or* CHARACTERS.

1. THIS ($^3\surd\ a \times b + c$) ſignifies, that the Product of *a* and *b* added to *c* is to have its Cube Root extracted. The ſame of the Square Root.

2. This ($\surd\ a \times b + c - d$) ſhews, that after the Quantity *d* is taken out of the Product of *a* and *b* more the Sum of *c*, the Remainder is to have its Square Root extracted.

3. This long Daſh (——) is often uſed to link or couple Quantities together for the better reading or underſtanding them. Beſides they are differently ex-

preſſed to what they are when it is wanting. Thus, $a \times b + c + d$ has quite a different Signification to what it has with the Daſh over it thus, $a \times \overline{b + c + d}$; for $a \times \overline{b + c + d}$, ſignifies that the Quantity a is to be multiplied by the Sum of b, c, and d; whereas without the Daſh, it would ſignify only that the Sum of the Quantities c and d is to be added to the Product of a into b This will be beſt underſtood by Numbers.

☞ Let 6 repreſent a, 4 b, 8 c, and 12 d. Now $6 \times \overline{4 + 8 + 12} = 144$; but $6 \times 4 + 8 + 12$, will be but 44. So that the Difference on account of the Daſh is 100. *Note further*,

This Daſh is often joined to the Tail of the radi~~ant~~cal Sign, or the Sign is continued longer, which is the ſame; and accordingly, as far as the Daſh is extended it has a different Signification.

4. Thus $\sqrt{m + \frac{bb}{4} + dd} - c)$ ſignifies, that the Quantity c is to be taken out of the Square Root of $m + \frac{bb}{4} + dd$.

5. But $\sqrt{m + \frac{bb}{4}} + dd - c)$ ſignifies, that only the Square Root of $m + \frac{bb}{4}$ is to be extracted, and then the Difference between the Quantities $dd - c$ to be added to it. The ſame for the Cube Root.

6. This Character ($\pm$) ſignifies *more or leſs ſuch a Quantity*, and is uſed often in Extraction of Roots, compleating of Squares, *&c.*

7. ☞ Figures are frequently ſet over Quantities, to ſhew how often they are expreſſed, and to ſave the Trouble of repeating or ſetting down the Letters ſo often. Thus, b^4 ſignifies the ſame as if the Quantity b was written, expreſs'd, or ſet down four Times, thus $bbbb$. So alſo x^8 is $xxxxxxxx$.

CHAP. II.

DIALOGUE V.

Between PHILOMATHES *and* TYRUNCULUS, *concerning* ADDITION, SUBTRACTION, MULTIPLICATION, *and* DIVISION *of* ALGEBRA.

Phi. ——— (*Coming to viſit* Tyrunculus.)
Tyr. ——— (*Standing at his Door.*)

Phi. GOOD Morrow to you, *Tyrunculus.*

Tyr. Kind *Philomathes*, your Servant; if I may be ſo free, where are you walking this Morning?

Phi. Why, *Tyrunculus*, to ſay the Truth, I came only for to ſpeak to you; it is ſome Time ago (if you remember) ſince I gave you a Paper, and I expected before now that you would have come for freſh Inſtructions and Examples; but your ſtaying ſo long began to make me think that you had given over the Thoughts of *Algebra* again, and had neglected to learn the *Signs* and *Characters* you had of me for that Purpoſe.

Tyr. I am ſorry I have given you Occaſion to think ſo; but the Reaſon of my not being with you before, is becauſe I have had ſome particular Buſineſs upon my

Hands, and *that* you know muſt be minded: However, I have learnt them perfectly by Heart, and know the Meaning of them very well.

Phi. I am glad of it; and you intend, I hope, to apply them to Practice; do you not?

Tyr. That I do, as far as Leiſure from more material Things will allow of.

Phi. Well then, are you at Liberty to go Home with me now, I am no Way engaged?

Tyr. If you inſiſt upon it, I will; but I had much rather you would ſpend an Hour or two with me now you are here; you ſhall be heartily welcome to ſuch Entertainment as my little Houſe affords, and I ſhall eſteem it as an Inſtance of your Kindneſs. What ſay you?

Phi. I heartily thank you, *Tyrunculus.* Entertainment by Way of eating and drinking I regard not, any further than to ſatisfy the real Wants of Nature: It is the Converſation I value, and had rather pleaſe my Mind than my Appetite; therefore upon Promiſe that you will not put yourſelf to any Trouble, nor provide for me any other than *that* which you intended for yourſelf had I not dropt in, I will ſpend a few Hours with you.

Tyr. Upon Honour I will not.

Phi. Come then, *Tyrunculus*, let us be doing.

Tyr. With all my Heart; and pray what is the firſt Thing in *Algebra* that I am to begin with?

Phi. As you underſtand *Vulgar Fractions*, and know the *Signs* and *Characters* you ſay, the very firſt Thing that I ſhew you will be *Addition.*

SECT. I.

ADDITION *of* ALGEBRA.

Tyr. HOW is *Addition* of *Algebra* performed?

Phi. The ſame as common *Addition*, provided the Signs be both *affirmative* or both *negative*; as you will ſoon find by the four following Caſes.

CASE I.

Of Simple Quantities, *or* Integers *having the ſame Sign.*

When the Quantities to be added have the ſame Sign, (viz. *both* + *or both* —) *then add all the* Co-efficients *or Numbers together, (if any there be) and place the Quantity after them, with the ſame Sign alſo before them.*

$$\begin{array}{lrrr} & \text{Ex. 1.} & \text{Ex. 2.} & \text{Ex. 3.} \\ \text{Add} & +\ a & +\ 2b & -\ 5d \\ & +\ a & +\ b & -\ d \\ & +\ a & +\ 4b & -\ 7d \\ \hline \text{Sum} & +\ 3a & +\ 7b & -\ 13d \end{array}$$

$$\begin{array}{lrr} & \text{Ex. 4.} & \text{Ex. 5.} \\ & +\ 5aadd & -\ bcd \\ & +\ 4aadd & -\ bcd \\ & +\ 6aadd & -\ bcd \\ \hline \text{Sum} & +\ 15aadd & -\ 3bcd \end{array}$$

Do you underſtand theſe Examples?

Tyr. Yes, except the *(b)* that ſtands alone in *Example* 2. and *(— d)* in *Example* 3. for they have no Number or Co-efficient before them.

Phi. It is true they have not; but they are ſuppoſed (as all ſingle Quantities are) to have Unity or 1 placed before them, as you may ſee in *Example* 1. and *Example* 5. whoſe Sums are 3*a*, and — 3*bcd*.

Tyr. I underſtand you now very well.

Phi. You are then further to take Notice, that all Quantities that have not the *negative* Sign (—) placed before them, are ſuppoſed to have the *affirmative:* Or, in other Words, thus: When any Quantity has no Sign prefixed to it, it is then an *affirmative* Quantity; thus, *a* is the ſame as + *a*, and 4 *b* is + 4 *b*.

CASE 2.

Of SIMPLE INTEGERS *or* QUANTITIES, *having contrary Signs.*

When the given Quantities are alike, but have unlike Signs, then ſubtract the one Co-efficient *from the other, and place the* negative *or* affirmative *Sign to the Remainder, according to where the Exceſs lies; that is, if the* negative *Quantity has the greater* Co-efficient, *then the Remainder will be* negative *and muſt have the Sign* (—); *but if the* affirmative *be the larger, then will the Remainder have the Sign* (+) *and will be the Sum of the ſaid Quantities.*

Ex. 1.	Ex. 2.	Ex. 3.
$4\,bb$	$-7\,bb$	$14\,aabb$
$-3\,bb$	$3\,bb$	$-43\,aabb$
$+\ bb$	$-4\,bb$	$-29\,aabb$

Ex. 4.	Ex. 5.
$-\ abcd$	$42\,bbdd$
$9\,abcd$	$-42\,bbdd$
$+8\,abcd$	(00)*

Do you underſtand theſe Examples?

Tyr. I underſtand all but the 5*th*, for I cannot at preſent conceive, that + 42 added to — 42 can be equal to Nothing; I ſhould think rather, that ſubtracting them they would be equal to nothing.

Phi. That is your Miſtake, for their Difference is — 84, (as you will ſee *Caſe* 2 in *Subtraction*;) becauſe the *negative* Sign makes void the *affirmative.*

Tyr. I aſk Pardon, but I do not rightly apprehend it.

---

* Leſt the Learner ſhould miſtake the Remainder in this Example, and take it for ſome Quantity, I thought proper to put it in a Parentheſis, to ſignify it for a Cypher only, and not for any Quantity. See *Dial. Obſervation* 2.

Phi. I think you are a little dull now. Do you not remember that I told you, (*Sect.* 3. *Dialogue* 4.) that this (—) ſignifies a Want or Deficiency, ſo many Times *leſs* than Nothing as the Figures after it expreſs?

Tyr. Yes, I do.

☞ *Phi.* Obſerve then, Suppoſe that you ſtood indebted to a Perſon 42£. and had no Effects of any Sort to pay the Debt, then it is plain you would be 42 Times worſe than Nothing; that is, have 42 Times leſs than a real Property of your own: Now ſuppoſe a Friend ſhould give you 42£. to pay off the Debt, and you do ſo, ſtill it is plain you would have Nothing in Hand to begin the World again with: Conſequently then $+ 42$ added to $- 42 = *$ or 0.

Tyr. I am very thankful, *Philomathes*, for ſo plain a Demonſtration.

☞ So I perceive then, that a *negative* Quantity added to an *affirmative* one, is the ſame as two *affirmative* Quantities ſubtracted from each other. Are they not?

Phi. The very ſame. (See *Caſe* 2. *Ex.* 5. in *Subtraction.*)

Tyr. Thus far then I am pretty perfect; but how muſt I manage when the Quantities are many in Number, and have different Signs?

☞ *Phi.* Very eaſily. Firſt collect all the Quantities that have one and the ſame Sign into one Sum, ſo you will have two Quantities at laſt to be added as above.

Tyr. Pray add $14\,axx - 5\,axx + 8\,axx - axx - 41\,axx + 39\,axx$ together?

Phi. Obſerve then, I collect all the Quantities having one and the ſame Sign together, *viz.* $14\,axx + 8\,axx + 39\,axx$, and theſe are equal to $61\,axx$; then $-5\,axx - axx - 41\,axx = -47\,axx$; then $-47\,axx$ added to $+61\,axx$, as before directed, $= 14\,axx$ *Anſwer*, the Sum of all the Quantities. Now I will try you with a Queſtion.

EXAMPLE 7.

Add $4\,aaa + 9\,aaa - 12\,aaa - 4\,aaa - 19\,aaa + 14\,aaa + 6\,aaa - 9\,aaa$ together.

Tyr. Nothing eaſier. Firſt, $4\,aaa + 9\,aaa +$

$14aaa + 6aaa = +33aaa$; and $-12aaa - 4aaa - 19aaa - 9aaa = -44aaa$; *then* $-44aaa$ added to $+33aaa = -11aaa$ *Anſwer*.

Phi. You are right, but ſtill you have taken unneceſſary Trouble.

Tyr. Wherein?

☞ *Phi*. Why did not I tell you that the *negative* Sign $-$ deſtroys the *affirmative* $+$; therefore as you have $4aaa + 9aaa$, and alſo $-4aaa - 9aaa$ in the Queſtion, you needed not to have meddled with them at all, only add the reſt of the Quantities, and you will find them to be $+20aaa$ and $-31aaa$, whoſe Sum is $-11aaa$, as above.

Tyr. I was a little wanting indeed in this Reſpect.

Phi. To be ſure it ſaves Trouble; for ſuppoſe I were to add $abb + 6abb - 9abb - abb + 4abb - 6abb$ to $+9abb - 3abb$ together, I have only $-3abb$ to add to $+4abb$, (for the reſt deſtroy each other by contrary Signs) and their Sum is $+abb$, the Sum of all.

Tyr. I ſhall take Notice of it; but ſuppoſe the Quantities to be added are not alike, nor the Signs neither, how then?

Phi. You will ſee by the following *Caſe*.

CASE 3.

Of SIMPLE CONTRARY QUANTITIES.

When the Quantities to be added are unlike, whether they have Co-efficients *or not, ſet them one after another, without any Alteration of the Signs, and this will be the proper Sum.*

Tyr. This is eaſy indeed; then if I were to add $3bc + aa + g - d$ together, I imagine their Sum is the ſame; *viz.* $3bc + aa + g - d$: Is it not?

Phi. You are very right. See the following *Example*.

EXAMPLE 1.

$$\begin{array}{ll} & -5bc \\ & +4da \\ & -\ xx \\ & -4cg \\ \hline \text{Sum} & -5bc+4da-xx-4cg \end{array}$$

EXAMPLE 2.

$$\begin{array}{ll} & -57gb \\ & +\ 4m \\ & -\ ggg \\ & +17 \\ \hline \text{Sum} & -57gb+4m-ggg+17 \end{array}$$

CASE 4.

Of COMPOUND INTEGERS *or* QUANTITIES.

This depends upon the three preceding Caſes: For if the Quantities are alike, and have the ſame Sign, then add them together by Caſe 1. *but if they have contrary Signs, collect all ſuch together as have one and the ſame Sign, and ſubtract them from each other, ſetting the Sign where the Exceſs lies (according to* Caſe 2.) *be it + or —. But if the Quantities and Signs be both contrary, then (by* Caſe 3.) *ſet them one after another without altering any of the Signs, and you have the Total.*

Ex. 1. Ex. 2.

$$\begin{array}{ll} \text{Add} & 4x+b \\ & \ \ x+4b \\ \hline \text{Sum} & 5x+5b \end{array} \qquad \begin{array}{r} 14x-9 \\ -9x+14 \\ \hline +5x+5 \end{array}$$

EXAMPLE 3.

$$\begin{array}{ll} \text{Add} & 4bc+x-18 \\ & 9bb-xb+9 \\ \hline \text{Sum} & 4bc+x+9bb-xb-9 \end{array}$$

More EXAMPLES.

EXAMPLE 4.

$$\begin{array}{l}
4\,abcd + 6\,bc + 9\,abd - 9 \\
-\ abcd - 4\,bc - 5\,abd \\
+2\,abcd - 3\,bc + 2\,abd \\
-\ abcd + bc + abd + 7 \\
\hline
\text{Sum}\ \ 4\,abcd + 7\,abd - 2
\end{array}$$

EXAMPLE 5.

$$\begin{array}{l}
a + 4\,b + g + ee + 7\,xx + y \\
x + hh + nn + 4\,ff - 12\,y \\
-\ a - g + 4\,ee + 4\,hh - xx - ff \\
-\ 5\,ee - 3\,ff + 5\,y - nn + 5 - zz. \\
\hline
\text{Sum}\ 4\,b + 6\,xx + 5\,hh + x - 6\,y + 5 - zz
\end{array}$$

Tyr. You have made Uſe of all Caſes I ſee in theſe *Examples*.

Phi. I do it on Purpoſe to ſerve you; and do you think you are perfect in *Addition?* Look at *Example* 5. once more, if you be not.

Tyr. I think I am indeed, Sir.

Phi. If ſo, we will paſs directly to *Subtraction*.

SECT. II.

SUBTRACTION *of* ALGEBRA.

Tyr. I AM afraid *Subtraction* will puzzle me, for I remember I was more ſet in this Rule in *Vulgar Fractions* than in any of the other.

Phi. Be not at all diſcouraged, for you will preſently do it, I am ſenſible, if you take Care to mind the Rules; for I ſhall proceed the ſame as in *Addition*.

CASE 1.

Of SIMPLE INTEGERS, *having the ſame Sign.*

When you have any Quantity given to be ſubtracted from another, then change the Sign of the Quantity to be ſubtracted into the contrary Sign; that is, if it be + *make it* —, *and if* — *make it* +, *and then add them by the firſt* Caſe *of* Addition, *and you have the true Difference: Or rather, if the Quantities have one and the ſame Sign, then ſubtract the* Numbers *or* Co-efficients *as in common* Subtraction, *obſerving to place the Sign belonging to the Remainder.*

	Ex. 1.	Ex. 2.	Ex. 3.
From	$3a$	$-21b$	$8bcd$
Take	a	$-9b$	$3bcd$
Anſ.	$2a$	$-12b$	$5bcd$

EXAMPLE 4.

From $-24abd$

Take $-9abd$

Anſ. $-15abd$

CASE 2.

Of SIMPLE INTEGERS, *having contrary Signs.*

When the Quantities to be ſubtracted have the negative Sign, *and the other the* affirmative, *then add them together as in common* Addition, *and you have their true Difference; to which place the* affirmative Sign, *but if the Quantity to be ſubtracted have the* affirmative Sign, *and the top Quantity the* negative, *you are to add as before, but then place the* negative Sign *to their Sum, and it is the true Difference.*

	Ex. 1.	Ex. 2.	Ex. 3.
From	$4bb$	$-7bb$	$14aabb$
Take	$-3bb$	$+3bb$	$-43aabb$
Diff.	$+7bb$	$-10bb$	$57aabb$

	Ex. 4.	Ex. 5.
From	$-\,abcd$	$+\,42\,bbdd$
Take	$9\,abcd$	$-\,42\,bbdd$
Difference	$-\,10\,abcd$	$+\,84\,bbdd$

EXAMPLE 5. *varied.*

From	$42\,bbdd$	$+\,42\,aabb$	$-\,42\,bbdd$
Take	$42\,bbdd$	$-\,42\,aabb$	$+\,42\,bbdd$
Diff.	(o)	$+\,84\,aabb$	$-\,84\,bbdd$*

CASE 3.

Of COMPOUND QUANTITIES.

There is no Difference in the Work of theſe and ſimple Quantities, only obſerve to place the Sign between the Difference according as is required.

	Ex. 1.	Ex. 2.
From	$a+b$	$a-b$
Take	$a-b+9$	$-a+b-9$
Anſ.	$+\,2b-9$	$2a-2b+9$

EXAMPLE 3.

From	$7aa+b-7cc+14$
Take	$-aa-4b+2cc-10$
Anſ.	$+8aa+5b-9cc+24$

Tyr. Mighty pretty, and withal very eaſy. Pray how are unlike Quantities ſubtracted?

Phi. I will tell you directly.

CASE 4.

Of CONTRARY *or* UNLIKE QUANTITIES.

When you are to ſubtract unlike Quantities, let them be either ſimple *or* compound, *you have no more to do,* *but*

---

* See *Caſe* 2. Ex. 5. in *Addition.*

but only to ſet the Quantities to be ſubtracted directly after the Quantity you are to ſubtract from, firſt obſerving to change or alter their Signs; thus have you their true Difference.

EXAMPLE 1.

From $a + b - 5c$
Take $g - d + 9$, by changing Signs,

Anſ. $a + b - 5c - g + d - 9$

EXAMPLE 2.

From $42bc + 24d - gg$
Take $17b - 14xx - bb - 15$

Anſ. $42bc + 24d - gg - 17b + 14xx + bb + 15$

Tyr. Quite plain indeed: Pray what is next?

Phi. Now, *Tyrunculus*, we are come to *Multiplication.*

SECT. III.

MULTIPLICATION *of* ALGEBRA.

Tyr. I Think that *Subtraction* is eaſier than I thought for.

Phi. Nothing like Delight and Application, *Tyrunculus*, theſe make Things eaſy.

Tyr. It is very true; but I am afraid of *Multiplication.*

Phi. Moſt Learners dread a freſh Rule, though they wiſh too to be trying at it: But what ſhould be the Reaſon for ſuch Fear, ſince the ſame Care will conquer one as the other.

Tyr. Pray how is *Multiplication* performed?

Phi. There are three Things to be obſerved in this Rule, *viz.*

1. *When the Quantities have the ſame Sign.*
2. *When they have contrary Signs.*
3. *When they have Co-efficients or Factors.*

In all which Caſes you are carefully to obſerve that when the Signs are both *affirmative*, or both *negative*, the Product will be *affirmative*; but when one is *affirmative*, and the other *negative*, the Product will be *negative:* For $+ \times +$, or $- \times -$, produce $+$; but $+ \times -$, or $- \times +$, produce $-$.

CASE 1.

Of QUANTITIES *having the ſame Sign.*

If the Quantities have no Factors *or* Co-efficients, *then only place the Letters repreſenting ſuch Quantities, one by the Side of the other, and place the Sign + before them; but if they have* Factors, *multiply them as in common* Multiplication, *placing the Product before the Quantities.*

	Ex. 1.	Ex. 2.	Ex. 3.
Mult.	a	$4b$	$-ab$
by	b	c	$-d$
Product	ba	$4bc$	$+abd$

	Ex. 4.	Ex. 5.
Mult.	$-6x$	$14bb$
by	$-2g$	$2b$
Product	$+12xg$	$28bbb$

Tyr. This is eaſy enough, indeed.
Phi. Nothing eaſier, therefore we will paſs to *Caſe* 2.

CASE 2.

To MULTIPLY *Quantiſies having contrary Signs.*

	Ex. 1.	Ex. 2.	Ex. 3.
Mult.	b	ba	$-4bcd$
by	$-a$	$-b$	$+6b$
Product	$-ab$	$-bba$	$-24bbcd$

EXAM-

EXAMPLE 4.

$$\begin{array}{lr} \text{Multiply} & 8\,xxb \\ \text{by} & -\;4\,d \\ \hline \text{Product} & -\;32\,xxbd \end{array}$$

Tyr. There need no more Examples. Pray what is the next?

Phi. CASE 3.

Of COMPOUND QUANTITIES.

Multiply every particular Quantity in the Multiplicand, *by each Member or Part of the* Multiplier, *as you do in common* Multiplication; *then add them together by the Rule of* Addition, *and the Work is done.*

$$\begin{array}{lr} & \text{Ex. 1.} \\ \text{Mult.} & ab + c \\ \text{by} & d \\ \hline \text{Product} & abd + dc \end{array} \qquad \begin{array}{r} \text{Ex. 2.} \\ 4\,b - cd \\ 4\,b \\ \hline 16bb - 4\,cdb \end{array}$$

EXAMPLE 3.

$$\begin{array}{lr} \text{Mult.} & 9\,b - 8 + 5\,x \\ \text{by} & -\;3\,d \\ \hline \text{Product} & -\;27\,bd + 24\,d - 15\,xd \end{array}$$

More EXAMPLES *of* COMPOUND QUANTITIES.

$$\begin{array}{ll} & \text{EXAMPLE 4.} \\ \text{Mult.} & a + b \\ \text{by} & a + b \\ \hline & aa + ab \\ & \quad + ab + bb \\ \hline \text{Prod.} & aa + 2\,ab + bb \end{array} \qquad \begin{array}{l} \text{EXAMPLE 5.} \\ a - b \\ a - b \\ \hline aa - ab \\ \quad - ab + bb \\ \hline aa - 2\,ab + bb \end{array}$$

EXAM-

EXAMPLE 6

$$\begin{array}{rl} \text{Mult.} & a + b \\ \text{by} & a - b \\ \hline & aa + ab \\ & \qquad - ab - bb \\ \hline \text{Product} & aa \qquad - bb \end{array}$$

EXAMPLE 7.

$$\begin{array}{rl} \text{Mult.} & ab + cd - 6 \\ \text{by} & c + d \\ \hline & abc + ccd - 6c \\ & \qquad + abd + cdd - 6d \\ \hline \text{Prod.} & abc + ccd - 6c + abd + cdd - 6d \end{array}$$

EXAMPLE 8.

$$\begin{array}{rl} \text{Mult.} & 2b + cd - 4 \\ \text{by} & -7x + 3 \\ \hline & -14bx - 7cdx + 28x \\ & \qquad + 6b + 3cd - 12 \\ \hline \text{Prod.} & -14bx - 7cdx + 28x + 6b + 3cd - 12 \end{array}$$

EXAMPLE 9.

$$\begin{array}{rl} \text{Mult.} & a + b + c \\ \text{by} & a + b + c \\ \hline & aa + ab + ac \\ & \qquad + ab + bb + bc \\ & \qquad\qquad + ac + bc + cc \\ \hline \text{Prod.} & aa + 2ab + 2ac + 2bc + bb + cc \end{array}$$

EXAMPLE 10.

$$\begin{array}{rl}
\text{Mult.} & a + b + c \\
\text{by} & a - b - c \\
\hline
 & aa + ab + ac \\
 & \quad\;\; - ab - bb - bc \\
 & \qquad\qquad - ac - bc - cc \\
\hline
\text{Product} & aa - bb - 2\,bc - cc \\
\hline
\end{array}$$

Tyr. I underſtand the Examples very well; I ſee in this laſt Example, that $+ ab - ab$, $+ ac$ and $- ac$ deſtroy each other when you come to add them together.

Phi. Very well obſerved, *Tyrunculus*; this gives me Reaſon to hope you are perfect in what you have ſeen done.

Tyr. I hope ſo. But pray, *Philomathes*, let me aſk a Queſtion; I am not fully ſatisfied why + × — ſhould produce —, and that — × — ſhould produce +; have you no other Way to demonſtrate it but barely telling me ſo?

Phi. To be ſure, it is eaſily proved as follows: Suppoſe I take 4 Quantities, and make them equal to any 4 Numbers, *viz.* Suppoſe $b = 12$, $c = 4$, $d = 8$, *and* $f = 6$. Then place them in ſome ſuch like Order as follows:

$$\text{Subtract}\left\{\begin{matrix} b = 12 \\ c = 4 \end{matrix}\right. \text{ and } \left.\begin{matrix} d = 8 \\ f = 6 \end{matrix}\right\}\text{Subtract}$$

Then $b - c = 8$, and $d - f = 2$

Now $\overline{b - c} \times \overline{d - f} = 8 \times 2 = 16$, the Product of their correſpondent Numbers 8 and 2. Therefore $\overline{b - c} \times \overline{d - f}$ will be $= db - dc - fb + fc$, *viz.* $= 16$, if the Work be right.

PROOF.

$$\begin{array}{ll}
\text{First, } b \times d = 12 \times 8 = 96 & \rbrace\ \text{Add} \\
\text{And } \ c \times f = \ \ 4 \times 6 = 24 & \\
\hline
\text{Therefore } bd + cf \qquad = 120 &
\end{array}$$

Again,

$$\left.\begin{aligned} d \times c &= 8 \times 4 = 32 \\ f \times b &= 6 \times 12 = 72 \end{aligned}\right\} \text{Add}$$

That is, $dc + fb = 104$, which taken from 120, leaves 16; that is, $bd + cf - dc - fb = 120 - 104 = 16$, as above. Conſequently therefore + × — produces —, and + × +, or — × — produces +, Q. E. D.

Tyr. I cannot ſay I underſtand it by looking it over ſo quick, but I will conſider of it another Time. In the mean Time, pray tell me what is the Meaning of Q. E. D.?

Phi. They are often uſed in the *Mathematicks*, and ſet at the End of a Demonſtration or Proof of any Problem, and ſignify, that the Thing given is not only done, but demonſtrated and proved, and is thus read, Q. E. D. *which was to be proved.*

Tyr. I heartily thank you. Pray have you any Thing more to ſhew me in this Rule?

Phi. Nothing more, only leave a Sum for you to work at your Leiſure, which I would have you try at.

EXAMPLE II.

Mult. $3xx + 4bb - 2d$
by $2xx - 3bb - d$
Prod. $6x^4 - xxbb - 7dxx - 12b^4 + 2bbd + 2dd$.

SECT. IV.

DIVISION *of* ALGEBRA.

Tyr. I Fear you will find me but a dull Scholar at *Diviſion*; for as common *Diviſion* is harder than the other three Rules, I think of Courſe *Diviſion* of *Algebra* muſt.

Phi. I confeſs it is ſomething more difficult; but what of all this? *Labour overcomes all Things.* However, it is eaſier perhaps than you imagine.

Tyr. How is *Diviſion* performed?

☞ *Phi.* In Diviſion are three Caſes; in all which you are to obſerve, as in *Multiplication*, that if the Signs be alike in the *Diviſor* and *Dividend*, the *Quotient* will be *affirmative*; but if unlike, the *Quotient* muſt have the *negative* Sign.

CASE I.

Of Quantities *having the ſame Sign.*

If the Quantities have Co-efficients, *belonging to them, then divide the one by the other, as in common* Diviſion, *and place the Quantities in the Quotient. But if the Quantities have no* Co-efficients, *then ſet the* Dividend *a-top, and the* Diviſor *under it,* Fraction-wiſe, *and if you find the ſame Letters or Quantities in both, cancel or caſt away ſuch Letters, and the Remainder will be the true Quotient or Anſwer required. Thus, ſuppoſe I were to divide* dcb *by* db, *I ſet it thus* $\frac{dcb}{db}$; *and becauſe I find* db *both in the* Dividend *and the* Diviſor, *I expunge or cancel them, and then only* c *remains, which is the Quotient or Anſwer.*

Ex. 1.

$$\begin{array}{rl}\text{Divide} & \underline{bbdc} \\ \text{by} & bc\end{array}\quad \textit{Anſ. } bd.$$

Ex. 2.

$$\frac{abc}{da}\quad \textit{Anſ. } \frac{bc}{d}$$

EXAMPLE 3.

$$\begin{array}{rl}\text{Divide} & \underline{xxddg} \\ \text{by} & xbgd\end{array}\quad \textit{Anſ. } \frac{xd}{b}$$

$$\begin{array}{lrrr} & \text{Ex. 4.} & \text{Ex. 5.} & \text{Ex. 6.} \\ \text{Divide} & 15\ xb & -\ 64\ bd & -\ 4\ abc \\ \text{by} & 5\ b & -\ 8\ b & -\ 4\ abc \\ \hline \textit{Anſ.} & 3\ x & +\ 8\ d & +\ 1 \end{array}$$

CASE

CASE 2.

Of CONTRARY SIGNS *and* QUANTITIES.

Divide the Quantities *and* Co-efficients *as before, and to the Quotient annex the Sign* —, *and the Work is done.*

	Ex. 1.	Ex. 2.	Ex 3.
Divide	ab	$-15xb$	$64bdc$
by	$-a$	$+5b$	$-8bd$
Anſ.	$-b$	$-3x$	$-8c$
Proof	ab	$-15xb$	$64bcd$

EXAMPLE 4.

Divide	$4abc$
by	$-4abc$
Anſ.	-1
Proof	$4abc$

Note, *When the* Signs *and* Quantities *are quite unlike, and the* Co-efficients *cannot be divided, then ſet them over one another* Fraction-wiſe, *and you have the Anſwer.*

	Ex. 5.	Ex. 6.	Ex. 7	Ex. 8.
Divide	ab	$14abg$	$xxbd$	$15xn$
by	g	$5xx$	ac	$12ag$
	$\frac{ab}{g}$	$\frac{14abg}{5xx}$	$\frac{xxbd}{ac}$	$\frac{15xn}{12ag}$

Tyr. I underſtand it very well; but how do you divide *compound Quantities?*

Phi. The ſame as in *Multiplication*, by going through every Member of the *Dividend* and *Diviſor* according to the Order of the common *Diviſion*.

CASE 3.

Of COMPOUND QUANTITIES.

Rule 1.

Proceed in all Reſpects as before, due Regard being had to the Signs, *and you have your Deſire.*

Ex. 1.

$$\begin{array}{rl} \text{Divide} & ada + abcd \\ & da \\ \hline \textit{Anſ.} & a + bc \end{array}$$

Ex. 2.

$$\begin{array}{c} 12\,dc - 15\,dx + 4\,ca - 5xa \\ 3\,d + a \\ \hline 4\,c - 5x \end{array}$$

Tyr. Let me aſk you one Queſtion ?

EXAMPLE 3.

$$\begin{array}{rl} \text{Divide} & 4\,abc - 24\,aabb - 32\,bad \\ \text{by} & -\;4\,ab \\ \hline \textit{Phi. Anſ.} & -1\,c + 6\,ab + 8\,d \end{array}$$

Tyr. Very well; but pray is not *Diviſion* wrought ſometimes at large, or do no Queſtions require it ?

Phi. It is frequently ſo wrought, and I could have done the laſt Example ſo if I would.

Tyr. Pray do, it may perhaps give me a better Idea of *Diviſion* than I have at preſent.

Phi. Obſerve then.

2. *When you have many* Quantities, *then proceed as in* Diviſion *of the whole Numbers, by ſeeing how many Times the* Diviſor *is contained in the* Quantities *of the* Dividend, *placing it in the* Quotient; *then multiply the* Diviſor *by the* Quotient *and place it under the* Dividend, *and ſubtract it therefrom, and to the Remainder bring down the next* Quantity *or* Quantities *in the* Dividend, *and thus proceed till the whole Operation is performed.*

EXAMPLE 3. *The long Way.*

$-4ab)\ 4abc-24aabb-32bad\ (-1c+6ab+8d$ *Anſ.*

$+4abc$

0

$-24aabb$

$-24aabb$

0

$-32bad$

$-32bad$

0

Tyr. I like this very well, and it appears much plainer to me than the other. Pray give me one more Example?

Phi. Suppoſe it were required to divide $bx+bd+cx+cd-xe-de$ by $x+d$, it will ſtand as follows.

EXAMPLE 4.

$x+d)\ bx+bd+cx+cd-xe-de\ (b+c-e$ *Anſ.*

$bx+bd$

0

$cx+cd$

$cx+cd$

0

$-xe-de$

$-xe-de$

0

EXAMPLE 5.

$$
\begin{array}{l}
3x-6)\ 6xxxx - 96\ (2xxx+4xx+8x+16 \quad \textit{Anſ.} \\
\qquad\quad 6xxxx-12xxx^{*} \\
\qquad\quad \overline{\qquad 0 \quad +12xxx-96} \\
\qquad\qquad\qquad\ +12xxx-24xx \\
\qquad\qquad\qquad \overline{\qquad 0 \quad +24xx-96} \\
\qquad\qquad\qquad\qquad\quad +24xx-48x \\
\qquad\qquad\qquad\qquad \overline{\qquad 0 \quad 48x-96} \\
\qquad\qquad\qquad\qquad\qquad\qquad 48x-96 \\
\qquad\qquad\qquad\qquad\quad \overline{\qquad\qquad\quad 0 \qquad}
\end{array}
$$

Tyr. I like this quite well indeed; but I could not have thought that ſo ſhort a *Dividend* would have produced ſo many *Quantities* in the *Quotient.*

Phi. That is eaſy to perceive; becauſe $* -12xxx$ is not found in the *Dividend.* I change the Sign (which is ſubtracting it) and bring it down for a new *Dividend* or *Remainder*, and it will we $+12xxx$. I do the ſame alſo with $-24xx$ and $-48x$. See the

PROOF.

$$
\begin{array}{l}
2xxx+4xx+8x+16 \\
3x-6 \\
\hline
6xxxx+12xxx+24xx+48x \\
\qquad\quad -12xxx-24xx-48x-96 \\
\hline
6xxxx-96
\end{array}
$$

Tyr. 'Tis right, ſure enough.

Phi. I ſhall give you then but one Example more.

EXAMPLE 6.

$$\begin{array}{l}
\ddagger \\
5b-9)20ab+5xxb-60b+30bc-5bd+40b-36a-9xx+108-54c+9d-72 \\
\underline{20ab-36a^{*}}(4a+xx-12+6c-d+8\ \textit{Anſ.} \\
+5xxb-60b \\
\underline{+5xxb-9xx} \\
-60b+3cbc \\
\underline{-60b+108} \\
+30bc-5db \\
\underline{+3cbc-54c} \\
-5db+40b \\
\underline{-5db+9d} \\
+40b-72 \\
\underline{+4cb-72} \\
0
\end{array}$$

Tyr. I do not underſtand this, that —* $36a$ ſhould ſtand under $5xxb$: Pray how do you ſubtract them?

Phi. I do not not ſubtract it from $5xxb$, though it ſtands under, but from — ‡ $36a$ further on in the Dividend, and ſo on for the reſt.

Tyr. Then it matters not whether the Quantities ſtand under each other I imagine; does it?

Phi. Not at all, as long as you can but find the ſame Quantity any where in the Dividend; and you would do well in ſuch long Sums to make a particular Mark againſt ſuch Quantities as you have taken down or done with, as you do in common *Diviſion*.

Tyr. I'll remember it; but ſuppoſe there ſhould be any Remainder (for I imagine the Quantities will not always fall out even) how then do you manage it?

Phi. The ſame as in common *Diviſion*, by placing it over the Diviſor *Fraction-wiſe*. Thus, ſuppoſe I were to divide $xx - bb + xc$, by $x + b$; I find then the Quotient to be $x - b$, and there remains xc; which I place over the Diviſor, and the Anſwer ſtands thus, $x - b + \frac{xc}{x+b}$

See the Work.

$$\begin{array}{lll}
x+b)\; & xx+xc-bb & \left(x-b+\dfrac{xc}{x+b}\right. \\
 & \underline{xx+xb} & \\
 & \quad -xb-bb+xc & \\
 & \quad \underline{-xb-bb} & \\
 & \quad\; * \quad * \quad xc &
\end{array}$$

Tyr. Have you any Thing more to offer in *Diviſion*.

Phi. I think there is no Occaſion for any more Examples.

Tyr. Pray what comes next?

Phi. According to the Order of *Arithmetic* the Rule of *Proportion* ſhould follow, but I ſhall ſpeak of this under *Dialogue* 7. and ſhew you firſt the Nature of *Algebraic Fractions*; though one would think there is no great Occaſion, ſince I have been ſo particular in *Vulgar Fractions*, in which, if you are perfect, you cannot miſs to underſtand the *Algebraic*, which are done one and the ſame Way, only with Letters inſtead of Numbers; and this can be no great Difficulty, ſince Numbers are only repreſented by ſuch Letters, as may be ſeen in the following *Dialogue*.

DIALOGUE VI.

SECT. I.

Of ALGEBRAIC FRACTIONS, *and firſt of* REDUCTION.

CASE 1*.

To reduce a mixt Quantity to an improper ALGEBRAIC FRACTION.

MULTIPLY *the whole Quantity by the Denominator of the* Algebraic Fraction, *and to the Product add the Numerator.*

EXAMPLE 1.

Reduce $a\frac{b}{x}$ *to an* improper Fraction.

$$\begin{array}{c} a\frac{b}{x} \\ \hline \frac{xa+b}{x}\ \textit{Anſ.} \end{array}$$

EXAMPLE 2.

Reduce $a+b\frac{c}{d}$ *to an* Improper Fraction.

$$\textit{Anſ.}\ a+b\frac{c}{d}=\frac{da+db+c}{d}$$

Tyr. I have ſeen ſeveral Books of *Algebraic Fractions*, but I do not remember any ſuch Examples as theſe: Are they neceſſary?

Phi. Certainly they are, and that you will ſee if you do but try the ſame by any Figures you pleaſe to make equal to the Quantities; and this will be ſome Help to you, and give you a Notion of an Equation.

---

* Compare this with *Caſe* 1. in *Vulgar Fractions.*

In *Example* 1. let $a = 5$, $b = 3$, $x = 6$, then

$$a\frac{b}{x} = 5\frac{3}{6}$$

$$\frac{xa+b}{x} = \frac{33}{6}\ \textit{Anſ.}\ \text{or rather}\ \frac{30+3}{6} = \frac{33}{6}.$$

For $x \times a = 6 \times 5 = 30$; $+ b = 3$, Q. E. D.

Tyr. I think it is neceſſary indeed, as you ſay.

Phi. And the Beauty of it is, the very next Caſe proves it; for I ſhall take the ſame two Examples.

CASE 2*.

To reduce an IMPROPER FRACTION *to a* MIXT QUANTITY.

This is only the Reverſe of the former, for you have no more to do but to divide the Numerator by the Denominator, and it is done.

EXAMPLE 1.

Reduce $\frac{xa + b}{x}$ *to its* equivalent mixt Numbers.

$$\begin{array}{l} x)\ xa + b\ (a\frac{b}{x}\ \textit{Anſ.} \\ \quad\ \ xa \\ \quad\ \ \overline{0 + b} \end{array}$$

EXAM-

---

* Compare this with *Caſe* 2. in *Vulgar Fractions.*

EXAMPLE 2.

Reduce $\frac{da + db + c}{d}$ *to its* equivalent mixt Numbers.

$$\begin{array}{l} d)\; da + db + c\,(a + b\tfrac{c}{d}\ \textit{Anſ.} \\ da \\ \hline 0 + db \\ db \\ \hline 0 + c \end{array}$$

CASE 3*.

To reduce a whole QUANTITY *to an* ALGEBRAIC FRACTION.

Multiply the given Quantity by any other Quantity, and place the Product for a Numerator, and the Quantity you multiplied by for a Denominator, and it is done.

EXAMPLE 1.

Reduce b *to an* Algebraic Fraction, *having* dc *for its Denominator.*

$b \times dc = bdc.$ *Anſ.* $\frac{bdc}{dc} = b$ by the next *Caſe.*

EXAMPLE 2.

Reduce a *to an* Algebraic Fraction, *having* x + f *for its Denominator.*

Firſt $a \times \overline{x + f} = ax + af.$ *Anſ.* $\frac{ax + af}{x + f}.$ Do you underſtand it?

Tyr. I cannot miſs it; you need not give any more Examples.

Phi. I ſhall not; but you ſhall ſee how that this Caſe is only the Reverſe of the next. We ſhall take the ſame two Examples.

---

* Compare this with *Caſe* 3. in *Vulgar Fractions.*

CASE 4[*].

To abbreviate an ALGEBRAIC FRACTION.

Divide the Numerator by the Denominator, that is, expunge or caſt away ſuch Quantities as are found in both, and you have your Deſire.

EXAMPLE 1.

Reduce $\frac{bdc}{dc}$ *to its loweſt Terms.*

$\frac{bdc}{dc} \div$ (divided by) $dc = b$. See *Example* 1. in laſt *Caſe*.

EXAMPLE 2.

Reduce $\frac{ax + af}{x + f}$ *to its loweſt Terms.*

$\frac{ax + af}{x + f} \div \overline{x + f} = a$. See *Example* 2. laſt *Caſe*.

So alſo $\frac{20\,aab}{60\,ba} + \frac{8\,bbd}{40bdg} = \frac{1\,a}{3} + \frac{1\,b}{5g}$.

☞ As *Fractions* are abbreviated by *Diviſion*, it is often cuſtomary to put Unity under ſuch Abbreviations when the Denominator is caſt away; that is, if the Anſwer be a whole Quantity, put the Figure 1 under it: Thus $a = \frac{a}{1}$; and $a + x + 4$, expreſſed like a *Fraction*, is, $\frac{a + x + 4}{1}$.

CASE 5[†].

To reduce Quantities of unequal Denominators to ALGEBRAIC FRACTIONS, *having a common Denominator.*

Multiply all the Denominators continually for a common Denominator, and every Numerator into all the Denominators except its own, which ſhall be new Numerators.

---

[*] Compare this with *Caſe* 4. in *Vulgar Fractions.*

[†] Compare this with *Caſe* 7. in *Vulgar Fractions.*

Tyr. I think I can do this directly.

Phi. No Doubt of it, for it is the ſame as *Caſe* 7. in *Reduction* of *Vulgar Fractions*.

EXAMPLE 1.

Reduce $\frac{a}{b}$, $\frac{c}{d}$, *and* $\frac{e}{f}$, *to* Fractions *having a common Denominator*.

Tyr. Firſt then, $b \times d \times f = bdf$ for a common Denominator. Then, $a \times d \times f = adf$ N. N. Again, $c \times b \times f = cbf$ N. N. And laſtly, $e \times d \times b = edb$ N. N. So are *adf*, *cbf*, *edb*, new Numerators to be placed over the common Denominator *bfd*, and will ſtand as follows.

$$\text{Anſ.}\ \left.\begin{array}{l} \frac{adf}{bdf} = \frac{a}{b} \\ \frac{cbf}{bdf} = \frac{c}{d} \\ \frac{edb}{bdf} = \frac{e}{f} \end{array}\right\} \text{by } \textit{Caſe}\ 4.$$

Phi. You are very right, *Tyrunculus*, and I am glad to ſee you ſo tractable. You ſee therefore that the Order of *Algebraic Fractions* is the ſame as of *Vulgar Fractions*.

Tyr. Yes, I perceive it, and I find your Words true now, that to underſtand *Vulgar Fractions* well ſaves a great deal of Trouble, that muſt unavoidably happen to thoſe that are ignorant of them. Pray what comes next?

Phi. I have here ſhewn you three Caſes more than are in general taken Notice of, that you might ſee the Relation that *Algebraic Fractions* bear to *Vulgar*. I ſhall therefore ſhew you now how to add them together.

SECT. II.

SECT. II.

ADDITION *of* ALGEBRAIC FRACTIONS.

Tyr. AS *Reduction* of *Algebraic Fractions* is like that of *Vulgar Fractions*, I imagine that *Addition* is done much after the ſame Manner alſo: Is it not?

Phi. The very ſame: You cannot miſs. Come, add $\frac{b}{x}$, $\frac{c}{x}$, and $\frac{d}{x}$ together.

Tyr. Becauſe the Quantities have a common Denominator, I only add the Numerators, *viz.* $b + c + d$, under which I place the common Denominator, x, and their Sum is $\frac{b + c + d}{x}$

Phi. Very right.

EXAMPLE 2.

Add $\frac{b}{x}$, $\frac{c}{d}$, *and* $\frac{f}{a}$ *together.*

Tyr. Firſt $x \times d \times a = xda$ for a C. D. and $b \times d \times a = bda$ N. N. Again, $c \times x \times a = cxa$ N. N. And laſtly, $f \times d \times x = fdx$ N. N. which I add as under:

$$\text{N. Numerators} \left.\begin{matrix} bda \\ cxa \\ fdx \end{matrix}\right\} \text{add}$$

$$\textit{Anſ.} \quad \frac{bda + cxa + fdx}{xda}$$

Or if you ſet down all the new Numerators over their common Denominators, and abbreviate them by *Caſe* the 4*th*, you will ſee they are the ſame, and will be equivalent to the *Fraction* given. Thus, $\frac{bda}{xda} = \frac{b}{x}$; $\frac{cxa}{xda} = \frac{c}{d}$; and $\frac{fdx}{xda} = \frac{f}{a}$.

Now I will aſk you one, if you pleaſe, with *mixt* Numbers.

Phi. With all my Heart.

EXAM-

EXAMPLE 3.

Tyr. *Add.* $4x\frac{b}{g}$ *and* $cd\frac{a}{f}$ *together.*

Phi. The Quantities not being alike, I add them only by the Sign +, and add the *Fractions* to them alſo by the ſame Sign as they ſtand. Thus, their Sum is $4x\frac{b}{g} + cd\frac{a}{f}$. Or otherwiſe thus, $4x + cd + \frac{bf + ag}{gf}$. Or, $\frac{4xg+b}{g} + \frac{cdf+a}{f}$.

Tyr. If it were not too much Trouble, *Philomathes*, I ſhould be glad you would demonſtrate this a little plainer to me.

Phi. That I'll do three Ways.

1. As the Quantities are unlike, I only place them one after the other, (according to *Caſe* 3. *Dialogue* 5.) and their Sum is $4x\frac{b}{g} + cd\frac{a}{f}$. Or,

2. By reducing the *mixt Numbers* to *improper Fractions*, their Sum is $\frac{4xg + b}{g} + \frac{cdf + a.}{f}$.

3. Reduce the *Fractions* $\frac{b}{g} + \frac{a}{f}$ to a common Denominator, you will have $\frac{bf + ag}{gf}$ to which add the whole Quantities, and you will have $4x + cd + \frac{bf + ag}{gf}$.

But however, I will even do more than you deſired, you ſhall ſee the *numerical* Proof. Let $x = 4$, then will $4x = 16$, and let the *Fraction* $\frac{b}{g}$ be $= \frac{1}{2}$. Make $c = 4$, and $d = 3$, then will $cd = 4 \times 3 = 12$, and let the *Fraction* $\frac{a}{f}$ be $= \frac{2}{3}$. Now at your Leiſure add $16\frac{1}{2}$, to $12\frac{2}{3}$, you will have $\frac{175}{6} = 29\frac{1}{6} =$ the *improper Fraction* $\frac{4xg+b}{g}+\frac{cdf+a}{f}$ which is in Numbers $\frac{32+1}{2}+\frac{36+2}{3}=\frac{175}{6},=29\frac{1}{6}$ as before Q. E. D. Does this appear plain to you?

Tyr. You lay me under the greateſt Obligations, *Philomathes*; it appears quite eaſy to me indeed.

Phi. I am glad of it, then you are qualified for *Subtraction*, and there is no Occaſion to dwell any longer upon *Addition*.

SECT. III.

SUBTRACTION *of* ALGEBRAIC FRACTIONS.

Tyr. HOW do you perform *Subtraction?*

Phi. By one general Rule, *viz.* If the *Fractions* have a common Denominator, ſubtract the Numerators, by placing the Sign (—) before that which is to be ſubtracted, and place the Difference over the common Denominator; and if they have different Denominators, reduce them (by *Caſe* 5. *Sect.* 1. of this *Dialogue*) to a common Denominator, and then ſubtract the Numerators as in *Subtraction* of *Algebraic Integers*.

EXAMPLE 1.

$$\text{From } \frac{a}{b} \text{ take } \frac{d}{b}$$

$$\text{Anſ. } \frac{a-d}{b}$$

Ex. 2.

$$\begin{array}{ll} \text{From} & \frac{x+b}{f+g} \\ \text{Take} & \frac{3b-x}{f+g} \\ \text{Difference} & \frac{2x-2b}{f+g} \end{array}$$

Ex. 3.

$$\begin{array}{l} \frac{12x+c+b}{ad+g} \\ \frac{2x+b-c}{ad+g} \\ \frac{10x+2c}{ad+g} \end{array}$$

Tyr. I muſt confeſs I do not apprehend theſe two laſt Examples.

Phi. You do well to ſay ſo, for they are not to be underſtood by every Learner at firſt Sight. Obſerve then the Numerators of *Example* 2 are $x + b$, and $3b - x$. Now (as before directed) I change the Sign of the *Fraction* to be ſubtracted, and then it will be $-3b + x$, which added (for this you muſt remember to do when the Signs are changed) to $x + b$, makes $2x - 2b$ for the Difference. And thus you muſt proceed with *Example* 3. always remembering that the *negative* and *affirmative* Sign before the ſame Quantity deſtroy each other, and you will find that $10x + 2c$ remains. The ſame is to be obſerved if the *Fractions* have not a common Denominator after they are once reduced to it. And thus much for *Subtraction*, provided you underſtand it.

Tyr. I thank you for this freſh Inſtruction; I am now pretty well grounded in this Rule, I believe.

SECT. IV.

MULTIPLICATION *of* ALGEBRAIC FRACTIONS.

Phi. WELL, *Tyrunculus*, what think you of *Multiplication?*

Tyr. I think I can work it without ſhewing, if I am not miſtaken.

Phi. That is right, *Tyrunculus*, I love to ſee you bold and courageous in every new Undertaking: Come then,

Multiply $\frac{a}{b}$ *by* $\frac{c}{d}$.

Tyr. If I remember, I am only to multiply the Numerators together for a new Numerator, and the Denominators for a new Denominator. Thus, $a \times c = ac$, and $b \times d = bd$: *Anſ.* $\frac{ac}{bd}$

Phi. You are very right. Now multiply $\frac{4b + c}{g + a}$ by $\frac{3d}{9x}$

Tyr. Firſt, $\overline{4b + c} \times 3d = 12bd + 3cd$ for a N. N. and $\overline{g + a} \times 9x = 9xg + 9xa$ for a N. D. So will the *Anſwer* be $\frac{12bd + 3cd}{9xg + 9xa}$

Phi. Very well done, *Tyrunculus*, indeed.

Tyr. Now, Sir, give me Leave to aſk you one?

EXAMPLE 3.

Multiply $\frac{12ax + b - 25x}{x}$ by $2b + 6x$.

Phi. Why, *Tyrunculus*, the Multiplier being a *whole Number*, I make a *Fraction of it*, and it will be

Mult. $\frac{12ax + b - 25x}{x}$ by $\frac{2b + 6x}{1}$.

Then multiplying the Numerators and Denominators together, I have

Anſ. $\frac{24abx + 2bb - 50bx + 72axx + 6bx - 150xx}{x}$. Or rather, it is $24ab - 44b + 72ax - 150x + \frac{2bb}{x}$ *abbreviate*. Do you underſtand it.

Tyr. Yes, very well, except the abbreviated Anſwer.

Phi. Pray look at *Caſe* the 4th in *Reduction*. You ſee as I have x for the Denominator, I caſt away or cancel x alſo in every Part of the Numerator; and as there is $-50bx + 6bx$, this will be $-44bx$; then cancelling x, it is $-44b$, and x at laſt is ſet under the Quantity $2abb$ *Fraction-wiſe*, becauſe x is not found in it.

Tyr. I heartily thank you, *Philomathes*, for this Information.

Phi. EXAMPLE 4.

Mult. $b\frac{c}{d}$

by a

$$\frac{dba + ca}{a}$$ *Anſ.*

EXAMPLE 5.

Mult. $4\,bx\frac{c}{d}$

by $\frac{5b}{9x}$

$$\frac{20bbxd + 5cb}{9dx}$$ *Anſ.*

Reduce the *mixt* Numbers to *improper Fractions*, and proceed as before, you will have the Anſwers as above; which I beg you will try at your Leiſure.

Tyr. I thank you, *Philomathes*; I will try it directly. Pray have you any Thing further to ſay of this Rule?

Phi. Nothing but this, that when it is required to multiply any *Fraction* by its Denominator, then caſt away the Denominator, and the Numerator alone is the Anſwer. Thus, $\frac{d}{x} \times x$ gives d: For $\frac{d}{x}\times\frac{x}{1}=\frac{dx}{1x}$ or $\frac{dx}{x}=d$. So alſo, $\frac{9b+7dx}{4x+d}\times\overline{4x+d}$, gives $9b+7dx$ *Anſ.* = to the Numerator. This is evident; for ſuppoſe $\frac{3}{7}$ to be multiplied by 7, it is $\frac{3}{7}\times\frac{7}{1}=\frac{21}{7}=3$ the Numerator of the original *Fraction* $\frac{3}{7}$, which in Fact is, only multiplying its Numerator by Unity or 1. And now, *Tyrunculus*, we will proceed to *Diviſion*.

SECT. V.

DIVISION *of* ALGEBRAIC FRACTIONS.

Tyr. I Dare ſay I ſhall not be able to work *Diviſion* without ſhewing.

Phi. Poh! You are now going to be dead-hearted again, and without Cauſe; and I had much rather find you as bold as you were in *Multiplication*. Conſider,

ſider, *Tyrunculus*, that every Learner may be compared to a young unexperienced Soldier; and though we will not call *Arithmetic* his Enemy, yet he has got many Skirmiſhes to go through, and muſt not only *fight*, but that *valiantly* too, to overcome them; for a Field is ſeldom won by Cowardice: Beſides, *Tyrunculus*, I have hitherto furniſhed you with Weapons proper for ſuch Engagements as you have met with, and I ſhall take Care to provide you with others for every freſh Attack; and do you but learn to handle them well, and you need not fear but you will always overcome.

Tyr. You lay me under the higheſt Obligations to love and thank you, for being ſo careful of me. Pray then how is *Diviſion* performed?

Phi. The ſame as in *Vulgar Fractions*. *Multiply the Numerator of the Dividend into the Denominator of the Diviſor for a new Numerator, and the Denominator of the Dividend into the Numerator of the Diviſor for a new Denominator.*

	Ex. 1.	Ex. 2.	Ex. 3.
Divide	$\frac{b}{g}$ by $\frac{d}{c}$	$\frac{14b}{g}$ by $\frac{2}{a}$	$\frac{4xx}{g}$ by $\frac{b}{4c}$
Anſ.	$\frac{bc}{gd}$	$\frac{14ba}{2g}=\frac{7ba}{g}$	$\frac{16xxc}{bg}$

☞ *Tyr*. I underſtand it very well; but ſuppoſe the *Fractions* to have one and the ſame Denominator?

Phi. Then caſt away both the Denominators, and divide the Numerators only.

EXAMPLE 4.

Divide $\frac{ba+bx+ca+cx}{5gg+d}$ by $\frac{b+c}{5gg+d}$. Then it will be thus, Divide $ba+bx+ca+cx$

by $b+c)ba+ca$ $(a+x$ *Anſ.*

$$\begin{array}{rl} & \underline{} \\ 0 & bx+cx \\ & \underline{bx+cx} \\ & 0 \end{array}$$

Do you underſtand theſe Examples?

Tyr. Yes I do, quite well.

Phi. Where then is the Difficulty you ſo much apprehended?

Tyr. I muſt confeſs that I was a little fearful juſt now.

Phi. I know it, I could ſee it in your Countenance.

Tyr. You have been ſo kind, that I muſt confeſs I cannot value you too much; nor can I repay you but with Thanks.

Phi. I do it to ſerve you, as I obſerved, and if you receive Benefit by my Inſtructions, return the Thanks elſewhere. I only deſire you to be chearful and courageous, not timorous, for that will hinder you in your Purſuit.

Tyr. I will endeavour to follow your Advice in every Reſpect: but it is a dull Study for Learners for the firſt few Rules.

Phi. I own it is; but now you are paſt the worſt: You have in a great Meaſure drawn aſide the Maſk, and as ſoon as you are acquainted with the Rule of *Proportion*, and underſtand a little of *Equations*, (which you will ſoon do) it will then fall quite off, and you will with Pleaſure be raviſhed with the Beauty of its Face, and the Symmetry of its Parts.

Tyr. How long will it be before I come to the Rule of *Proportion?*

Phi. I ſhall ſhew you the Nature of it directly.

* * *

DIALOGUE VII.

SECT. I.

Of PROPORTION *in general.*

Tyr. WHAT do you mean by *Proportion?*

Phi. Proportion is the Relation, Reſpect or Quality, that Numbers or Quantities bear to each other, by a certain *Ratio*, *Reaſon*, *Analogy*, or *Compariſon*.

Tyr. Is not *Ratio* and *Proportion* then all one and the ſame?

Phi. They are often uſed and ſpoken of as one and the ſame Thing, but there is a Difference; for ſtrictly ſpeaking, *Ratio* is not *Proportion* itſelf; but ſhews the Difference of Numbers, by comparing one with the other; ſo that by it is ſeen the *Reaſon* or *Analogy* that the *Antecedent* bears to the *Conſequent*, by ſuch and ſuch Compariſon, and in Courſe the *Proportion* that the Numbers bear to each other.

Tyr. What do you mean by *Antecedent* and *Conſequent?*

Phi. In any two Numbers or Quantities, the firſt Term is called the *Antecedent*, the ſecond the *Conſequent*. Thus, 4, 8, *&c.* 4 is the *Antecedent*, and 8 the *Conſequent*; the *Ratio* is 4, becauſe 8 is 4 + 4, and the *Compariſon* is 2, becauſe 4 and 8 compared, one is twice the other. So alſo in any Series of Numbers, as 2, 6, 10, 14, 18, *&c.* 2 and 18 are the *Antecedent* and *Conſequent*, and all the others between them are both *Antecedents* and *Conſequents*.

Tyr. I underſtand you very well, but pray how many Sorts of *Proportion* are there?

Phi. There is, 1. *Disjunct Proportion*, or *the Rule of Three Direct*. 2. *Arithmetical Proportion*, *or Progreſſion*. 3. *Geometrical Proportion*, *or Progreſſion*. 4. *Duplicate Proportion*. 5. *Triplicate Proportion*. 6. *Harmonical Proportion*. And 7. *Contra-harmonical Proportion*, &c. But it will be ſufficient for our preſent Purpoſe to ſpeak of the firſt *Three* only, ſince the Knowledge of the others depends upon theſe.

S E C T. II.

Of DIRECT PROPORTION, *or the* RULE *of* THREE.

Tyr. WHAT have I more to do with *Proportion*, ſince I can work the *Rule of Three* very well?

Phi. That may be; and yet you may not rightly underſtand the Nature of it. A great many Perſons deceive themſelves in this, for though they can work a Queſtion according to the Order of the Rule itſelf, yet they are quite ignorant of the *Relation* or *Proportion* which one Number bears to another, notwithſtanding it is evidently known that one Half of the *Mathematics* depend upon it.

Tyr. I thought if I could but barely work the Rule it was enough, ('till you ſhewed me to the contrary in *Abbreviations*;) but ſince I ſee it is of ſuch excellent Uſe, I beg you would explain the Nature of it a little plainer to me.

Phi. I intend it; but before I give a Demonſtration, it will be quite neceſſary that you ſhould be well acquainted with the following Obſervations, and then the Demonſtration will appear quite plain to you: So that my Advice is, you would read them over again and again.

OBSERVATION 1.

Any three Numbers or Quantities being propounded, after you have found a fourth Number in Proportion, according to the Order of the *Rule of Three direct*, then the Proof of ſuch Work is eaſily diſcovered; for if it be done right, the Proportion will always hold thus;

As the *1ſt* : *2d* : : *3d* : *4th*

Or, as the *1ſt* : *3d* : : *2d* : *4th*. That is, the *1ſt* bears the ſame Proportion to the *3d*, as the *2d* does to the *4th*.

OBSERVATION 2.

The Product of the *1ſt* and 4th is equal to the Product of the *2d* and *3d*; that is, the Product of the *Extremes* is equal to the Product of the *Means*: For the 3d divided by the *1ſt*, is equal to the *4th* divided by the *2d*, *&c.*

OBSERVATION 3.

The 1ſt is equal to the Product of the *2d* and *3d*, divided by the *4th*.

OBSERVATION 4.

The *2d* is equal to the Product of the *1ſt* and *4th*, divided by the 3d.

OBSERVATION 5.

The 3*d* is equal to the Product of the 1*ſt* and 4*th*, divided by the 2*d*.

OBSERVATION 6.

The 4*th* is equal to the Product of the 2*d* and 3*d*, divided by the 1*ſt*.

Tyr. I could not have thought there had been ſuch Harmony in the Rule of *Proportion*. But pray explain theſe Obſervations to me by ſome Demonſtration?

Phi. I will both by Quantities and Numbers, which, if you mind, you cannot but underſtand it.

DEMONSTRATION.

Let the 4 Quantities x, b, c, and d, repreſent any 4 Numbers in direct Proportion, *viz.*

Let $x=2$, $b=4$, $c=12$, and $d=24$.

		Literally.		Numerically.	
	1	As $x : b :: c : d$	i.e.	As $2 : 4 :: 12 : 24$	Obſ. 1.
Or	2	As $x : c :: b : d$		As $2 : 12 :: 4 : 24$	
For	3	$xd = cb$		$2 \times 24 = 4 \times 12$	Obſervation 2.
And	4	$\frac{b}{x} = \frac{d}{c}$		$4 \div 2 = 24 \div 12$	
Again	5	$\frac{x}{b} = \frac{c}{d}$		$\frac{2}{4} = \frac{12}{24}$	
And	6	$\frac{x}{c} = \frac{b}{d}$		$\frac{2}{12} = \frac{4}{24}$	
Laſtly	7	$\frac{c}{x} = \frac{d}{b}$		$\frac{12}{2} = \frac{24}{4}$	

From the four laſt Steps, each Term is found as follows:

Term

$$\left.\begin{array}{c|c|l} 1 & 1 & x=\frac{bc}{d} \\ 2 & 2 & b=\frac{xd}{c} \\ 3 & 3 & c=\frac{xd}{b} \\ 4 & 4 & d=\frac{bc}{x} \end{array}\right\} \textit{i. e.} \left\{\begin{array}{r} 2=\frac{48}{24} \\ 4=\frac{48}{12} \\ 12=\frac{48}{4} \\ 24=\frac{48}{2} \end{array}\right\} \begin{array}{l} \textit{Obſervation } 3. \\ \textit{Obſervation } 4. \\ \textit{Obſervation } 5. \\ \textit{Obſervation } 6. \end{array}$$

I ſhould think, *Tyrunculus*, the *numerical* Work is ſo plain, that you cannot help underſtand the *literal*, ſince the Steps of one anſwer to the other.

Tyr. Nothing can be plainer indeed. But pray what do you mean by *Steps?* and what is their Uſe?

Phi. Thoſe are the Steps that ſtand in the Margin of the Work, numbered 1, 2, 3, *&c.* Their Uſe is to ſhew the gradual Proceeding of the Operation, that you go on *gradatim*, or by Degrees; that is, Step by Step.

Tyr. Have you any Thing further to ſay of *direct Proportion?*

Phi. I have nothing more to add but this, that when Quantities or Numbers are in a *direct Proportion*, they are alſo *Proportionals* by *Alteration*, *Inverſion*, *Diviſion*, *Converſion*, and *Compoſition*, &c. See *Euc.* 5. *Def.* 12, 13, *&c.*

SECT. IV.

Of ARITHMETICAL PROPORTION.

Tyr. WHAT is *Arithmetical Proportion?*

Phi. Numbers or Quantities are ſaid to be in *Arithmetical Proportion*, or *Progreſſion*, when they differ from one another by a certain *Ratio*, or the like Reaſon. Thus, 2, 6, 10, 14, 18, 22, *&c.* are Numbers in *Arithmetical Progreſſion*, becauſe they differ from one another by the like Reaſon, *viz.* by 4, which Difference is called the *Ratio.* So 1, 19, 37, 55, *&c.* differ from each other by the *Ratio* 18, as you may perceive; for $1+18=19$, $19+18=37$, $37+18=55$, *&c.* From hence will follow this Obſervation.

OBSERVATION 1.

Any 3 Numbers or Quantities in *Arithmetical Proportion*, the Double of the *Mean* (or middle Number) is equal to the Sum of the *Extremes.*

Numerical Demonſtration.

Let the 3 Numbers be 5, 13, and 21, whoſe *Ratio* or common Difference is 8; the Double of the *Mean* 13, is equal to the Sum of the *Extremes*, viz. 5 and 21.

Literal Demonſtration.

Let x be put for the firſt Term 5, and let e repreſent the *Ratio*, which is 8. Then will $x+e=13$ the *Mean*, and $x+2e=21$ the third Term.

PROOF.

$$\begin{array}{lll} & \text{Mean.} & \qquad\qquad \text{Extremes.} \\ \text{Add} & \left\{\begin{array}{l} x+e=13 \\ x+e=13 \end{array}\right. & \qquad\qquad \left.\begin{array}{l} x+2e=21 \\ x \qquad\;\; =\;\;5 \end{array}\right\} \text{Add} \\ \hline \end{array}$$

$$\text{M. doubled } 2x+2e=26=2x+2e=26 \text{ Sum of Ex.}$$

Tyr. This is mighty plainly demonſtrated indeed! But pray muſt I always put this Letter (*e*) to repreſent the *Ratio?*

Phi. This is at your Option; you may uſe any Lettres, provided you put one for the Terms, and another for the *Ratio*, or Difference of the Terms.

OBSER-

OBSERVATION 2.

Any 4 Numbers or Quantities in *Arithmetical Proportion*, either continued or diſcontinued and interrupted, the Sum of the *Means* is equal to the Sum of the *Extremes*.

Numerical Demonſtration.

Let the 4 Numbers in *Arithmetical Progreſſion* be 4, 16, 28, and 40, whoſe *Ratio* is 12; then it is plain that 16 + 28 the *Means*, is equal to 4 + 40 the *Extremes*.

Literally.

Let a repreſent the firſt Term 4, and put x for the *Ratio* 12; then will $a + x = 16$, one *Mean*, and $a + 2x = 28$, the other *Mean*, and $a + 3x = 40$, the laſt Term or *Extreme*.

PROOF.

Means.	Extremes.
$a + x = 16$	$a = 4$
$a + 2x = 28$	$a + 3x = 40$

Means $2a + 3x = 44 =$ *Extremes* $a + 3x = 44$.

N. B. It would be the ſame if the Numbers had been diſcontinued, provided the Interruption be between the 2*d* and 3*d* Term. Thus, ſuppoſe the 4 Numbers were 4, 16, 124, 136; then $4 + 136 = 16 + 124 = 140$. For there is the ſame *Ratio* between the 3*d* and 4*th*, as there is between the 1*ſt* and 2*d*, *viz.* 12.

Tyr. I heartily thank you, kind *Philomathes:* Have you any Thing further to add upon this?

Phi. I am not willing to leave any Thing out that may be ſerviceable; but I think I have ſaid enough upon this Rule for your preſent Occaſion. However, it may be expected I ſhould teach you to work ſome Queſtions, or at leaſt give you ſome Rules to work them by.

Tyr. I think that Mr. *Ward* (in his *Arithmetic, Page* 76) ſpeaks of twenty Theorems belonging to this Rule; but he has given Examples only of two of them, and the other eighteen I find in his *Algebra*, *Page* 186, but having no Rule for them, they are (I ſhould think) beyond the Reach of moſt Learners. I ſhould have liked he had given the Rule for finding them, though he had not done the Operation itſelf; becauſe by a plain Theorem, or Rule to work by, any aſſiduous Learner would know how to put Things in Practice that are not very difficult; but how ſhould he know when he has no Rule to go by, nor any Tutor at Hand?

Phi. Had he given you the Work of ſix Theorems with their Rules, you might with Eaſe have found out the reſt; as you will diſcover by the ſix following *Caſes*; the 2*d* and 5*th* of which will anſwer to his two in *Page* 74 of his Work.

CASE 1.

The Number or Places of Terms, and the Ratio or common Exceſs being given, to find the laſt Number.

Multiply the Number of Places leſs one, by the *Ratio* or common Exceſs; and to that Product add the firſt Number, and the Sum will be the laſt Number.

CASE 2.

The firſt and laſt Number (viz. *the Extremes) and the Number of Terms being given, to find the Aggregate or total Sum of all the Series.*

Add the firſt and laſt Numbers together, and multiply the Sum by Half the Number of Places, and you have the Total of all the Series added together. Or, in Caſe the Number of Places be odd, then add the firſt and laſt Numbers together, and multiply the Sum by the whole Number of Places, and divide that Product by 2, and you have the Aggregate or total Sum.

CASE 3.

The Extremes and Total given, to find the Number of Terms.

Add the *Extremes* together, and divide the Total by their Sum, and the Quotient will be equal to Half the Number of Places.

CASE 4.

The Total and Number of Terms given, to find the laſt Number.

Divide the Total by Half the Number of Places, or in Caſe the Terms be odd, divide double the Total by the Number of Terms, and the Quotient will be a Number; from which if you take the firſt Term, the Remainder will be the laſt Number.

CASE 5.

The Extremes and Number of Terms given, to find the Ratio or common Exceſs.

From the greater take the leſs *Extreme*, and the Remainder ſhall be a Dividend; then from the Number of Terms take Unity, (*viz.* 1) and the Remainder ſhall be a Diviſor; and the Quotient riſing from them ſhall be the *Ratio*, or common Difference of the Terms.

CASE 6.

The Extremes and common Exceſs given, to find the Number of Terms.

From the greater take the leſs *Extreme*, and divide the Remainder by the common Exceſs; then to the Quotient add Unity, or 1, and that Sum will be equal to the Number of Places.

Tyr. Theſe Rules are very plain indeed, they need no Example.

Phi. Example and Precept are beſt together, therefore I will give you an *Example* in *Caſe* the 2*d* and *Caſe* the 5*th*, and you will, no Doubt, do the reſt upon the firſt Trial.

EXAMPLE *of* CASE 5.

Let the Number of Places be 8, the *Extremes* 4 and 30, I demand the *Ratio*?

Firſt, $39 - 4 = 35$ Dividend, then $8 - 1 = 7$ the Diviſor, and $35 \div 7$ gives 5 the common Exceſs. Proof 4, 9, 14, 19, 24, 29, 34, 39.

Or literally thus:

Let $x = 4$ leſs Extreme, and $e = 39$ the greater, and $b =$ the Number of Terms; then will $\frac{e - x}{b - 1} = 5$, the *Ratio* as above.

Tyr. I underſtand it very well. Now give me one Example of *Caſe* the 2*d*.

Phi. I will.

EXAMPLE *of* CASE 2.

Let the Numbers be as above, *viz.* 4, 9, 14, 19, 24, 29, 34, 39. It is required to find the Aggregate or total Sum of all the Terms added together.

$$\begin{array}{rrl} \text{Firſt Number or } \textit{Extreme} & 4 & \\ \text{Laſt} & 39 & \\ \hline & 43 & \text{Sum} \\ \text{This} \times \tfrac{1}{2} \text{ Number of Terms, } \textit{viz.} & 4 & \\ \hline \text{Total} & 172 & \end{array}$$

Or literally thus:

Let x be the firſt, and e the laſt Number, and let b repreſent Half the Number of Terms.

$$\begin{array}{rl} \text{Then } x + e = & 43 \text{ as above} \\ b = & 4 \\ \hline \text{Total } bx + be = & 172 \text{ as above.} \end{array}$$

So that from hence you ſee another Rule to find the Total, *viz.* Multiply the leſs *Extreme* and the greater ſeparately by Half the Number of Terms, and add their Products together, it will be the Sum of all the Series. And thus, x, e, and b, may repreſent the *Extremes* and Half the Terms, be they ever ſo many, which you are carefully to obſerve.

Tyr. I like this very well, and I am ſure it is far from being hard.

Phi. I ſhall leave you a Queſtion to try at your Leiſure, to ſee if your Anſwer be like mine.

Three or four Men in Company were diſputing concerning the Diſtance one muſt go, and the Time it would take, to gather up Stones laid each a Yard aſunder for one Mile in Length, and bring each Stone back to the End he began at. A ſilly bragging Jockey (who had preſent a good Horſe) ſaid he could ride further than that in three Hours. A Sharper in Company, taking Advantage of his Folly, ſaid, he would venture him fifty Guineas he did not ride his Horſe ſo far in three Days; the Jockey unwarily conſents; the Wager is ſtaked, and he was to ſet out next Morning; but long before this he found it better to yield it loſt than make Trial of ſuch an Impoſſibility, it being 3099360 *Yards* = 1760 *Miles; which is* $586\frac{2}{3}$ *Miles a Day.*

Tyr. Surpriſing! I will try at it very ſhortly. Pray what comes next?

Phi. Geometrical Progreſſion.

SECT. IV.

Of GEOMETRICAL PROPORTION.

Tyr. WHAT is *Geometrical Proportion?*

Phi. Geometrical Proportion, or *Progreſſion*, is when Numbers or Quantities differ from each other by like *Ratio* or Reaſon, as in *Arithmetical Progreſſion*, only with this Difference, that in *Arithmetical Progreſſion* the *Ratio* is the Effect of *Addition*, but in *this* of *Multiplication*, by having one common Multiplier.

Tyr. Pleaſe to explain this more cleary to me?

Phi. Obſerve then, 2, 4, 8, 16, 32, 64, ÷ &c. are Numbers in *Geometrical Proportion*, and differ by double Reaſon the one from the other, the common Multiplier being 2. They are every one you ſee the Double of the preceding Number. So alſo 4, 12, 36, 108, &c. differ by triple Reaſon, each Term being three Times its preceding one. And 1, 4, 16, 64, 256, &c. differ by quadruple Reaſon.

Tyr. I underſtand you now perfectly well.

Phi. Then you are to obſerve as follows.

OBSERVATION 1.

Any three Numbers in *Geometrical Proportion*, the Product of the *Extremes* is equal to the Square of the *Mean*; that is, equal to the middle Term multiplied by or into itſelf.

Let the 3 Numbers be 4, 16, and 64. Here $4 \times 64 = 16 \times 16 = 256$.

Literal Demonſtration.

Let x repreſent the firſt Term or *Extreme*, and let e be put for the *Ratio*, then will xe be the *Mean*, and xee the laſt Term, or other *Extreme*; then will $x \times xee$ be $=$ the Square of the *Mean* xe, viz. $xxee$.

PROOF.

	Extremes.	Mean.
	$x = 4$	$xe = 16$
	$xee = 64$	$xe = 16$
Product	$xxee = 256$	$xxee = 256$

Tyr. I underſtand the Example very well.

Phi. Once more then obſerve.

OBSERVATION 2.

Any four Numbers or Quantities in ∺, either continued or interrupted (provided the Interruption be between the 2*d* and 3*d* Term) the Product of the *Means* is equal to the Product of the *Extremes*.

EXAMPLE.

Let the 4 Numbers be 5, 15, 26, and 78 interrupted; then $5 \times 78 = 15 \times 26 = 390$. It will be eaſy to prove the ſame literally as above.

OBSERVATION 3.

The *Ratio* of any Series of Numbers in ∺ continued, is found only by dividing any of the Conſequents by its Antecedent; that is, dividing any Number by the preceding Number.

OBSERVATION 4.

Whenever ſo many Numbers or Quantities differ by double Reaſon, and it is required to find the laſt Number of all, the general Way of moſt Perſons is to double the 1*ſt*, 2*d*, 3*d*, 4*th*, *&c.* Number, and ſo continue to do till they have doubled as often as there are Terms given. But,

☞ There is a better Way when the Places are a great many, for you have no Occaſion to double but a few of the Terms, and then multiply that Number into itſelf, and the Product will be the double of the Terms wanting one; which doubled gives the next Term, *&c.*

Tyr. This muſt be further explained to me, I do not apprehend it.

Phi. It is a little dark in Words only; but you'll underſtand it the Moment you ſee it done.

Suppoſe then a Series of Numbers in ∺ from 1 to 80 Places were given, which differ by double Reaſon, and it was required to find the laſt Number. Firſt double a few of them, ſuppoſing to the 5*th* Place (which may be done by the Head only) then ſquare this Number, it ſhall give you the 9*th* Term; which doubled, gives you the 10*th* Term; this ſquared gives the 19*th* Term; which multiplied by 2, gives the 20*th* Term; this ſquared gives the 39*th* Term; which into 2 gives the 40*th*; this into itſelf gives the 79*th*; and laſtly, this doubled gives the 80*th* or laſt Term.

Tyr. You need not demonſtrate it any further; but how ſhall I find the Sum or Total of all the Series?

Phi. Very eaſily, by either of the following Methods.

OBSERVATION 5.

To find the Sum of all the Series.

1. Multiply the laſt Term by the *Ratio*, or *common Exceſs*, and from the Product ſubtract the firſt Term; then divide the Remainder by the *Ratio* wanting 1, and it will give you the Sum of the Series. Or rather,

2. From the laſt Term take the firſt, and divide the Remainder by the *Ratio*, or *common Exceſs*, leſs Unity or 1; then multiply the Quotient by the *Ratio*, and to that Product add the firſt Number, and you will have the Sum of all the Series.

Tyr. I heartily thank you, kind *Philomathes*, for your Trouble.

Phi. I ſhall not give you any Examples at large, but only ſhew you that the Increaſe of Numbers in ∺ is beyond the Belief or Conception of People in general. Thus, a Horſe having 8 Nails in each Shoe, and being bought or ſold at only 1 Farthing the firſt Nail, and double the Price for the next till you come to the thirty-ſecond Nail, would amount to the Sum of 4473924*l.* 5*s.* 3*d.* $\frac{3}{4}$. And one Nail more would make it 8947848*l.* 10*s.* 7*d.* $\frac{3}{4}$. Thus alſo if a Farmer's Servant ſhould agree with his Maſter to ſerve him 20 Years, for one Grain of Wheat only the firſt Year, and 10 the next, and ſo to have 10 Times the Number every Year, he would have 11111111111111111111 Grains for his Service. Now allowing, according to the Standard, that 7680 Grains make a Pint, the Number of Buſhels will be 22605613425926 nearly; which, at ſo ſmall a Price as Half a Crown a Buſhel, will amount to 2825701678240*l.* And ſuppoſe a Ship to carry 1000 Loads for her Burthen, it would take 1000 Times more ſuch Ships in Number than the whole World can furniſh. Which according to the foregoing Rules you may try at your Leiſure.

Tyr. Surpriſing indeed! depend upon it I will try it for Curioſity ſake; but pray before you finiſh this Head give an Idea of *Harmonical Proportion.*

Phi. Harmonical or *Muſical Proportion* is, when in 3 Numbers given, the Difference of the 1*ſt* and 2*d* is to the Difference of the 2*d* and 3*d*, as the 1*ſt* is to the 3*d*; or when in 4 Numbers given, the Difference of the 1*ſt* and 2*d* is to the Difference of the 3*d* and 4*th*, as the 1*ſt* is to the 4*th*. And,

Contra harmonical Proportion is, when in 3 Numbers given, the Difference of the 1*ſt* and 2*d* is to the Difference of the 2*d* and 3*d*, as the 3*d* is to the 1*ſt*; and ſuch are 6, 10, and 12.

Tyr. I am obliged to you. Pray what is the next Thing we are to learn?

Phi. You are now come to *Equations*, and pray take the greateſt Care you poſſibly can; for the ſolving of *Algebraic* Queſtions depends upon the true Knowledge thereof.

Tyr. I will be diligent to obſerve what you ſay.

DIALOGUE VIII.

Of EQUATIONS.

SECT. I.

Of REDUCTION.

Tyr. WHAT do you mean by an *Equation?*

Phi. An *Equation* is an exact *Equality*, or the mutual Agreement of two or more Things when compared together. Thus, when a Pound *Sterling* is compared with Shillings, it is found equal to 20, and a Crown compared with Groats is equal to 15 ſuch Pieces; therefore there can be no *Equation* where there are not two Things at leaſt, becauſe there can be no *Analogy* or *Compariſon:* And when there are two Numbers or Quantities, or more, to be compared with each other, you will always find this Sign or Character (=) placed between them.

Demonſtration.

Suppoſe x to repreſent a £. *Sterling*, and d 240 Pence, its Equivalent; then it is evident that $x = d$. Again, Suppoſe g to repreſent 5 Shillings, and e 15 Groats, then will $g = e$. But ſuppoſe g to repreſent a Shilling only, and e one Groat only, then there muſt be Numbers before the Quantities to form an *Equation*; for whereas before g was equal to e, now here it will be $g = 3e$, or $5g = 15e$; *viz.* 1*s.* = 3 Groats, 5*s.* = 15 Groats.

Tyr. I underſtand the Demonſtration very well.

Phi. You are further to obſerve, *Tyrunculus*, that in every *Equation* there are two Parts; that Part which ſtands before the Sign is called the firſt Part, and that after it the ſecond.

EXAMPLE.

Suppoſe $x = 4b + c$; then is x on the firſt Part equal to 4 Times the Quantity repreſented by b on the ſecond Part, together with the Quantity repreſented by c, added to it.

Tyr. Pray how are *Equations* formed?

Phi. This is a Queſtion that I cannot anſwer as yet to your Underſtanding; but you may learn thus far, that when one or more Letters, repreſenting any known Quantity, are found on the ſame Side of the *Equation* with other Quantities that repreſent unknown Quantities; then they muſt be ſo managed as to be brought on the other Side of the *Equation*; ſo that one Side of the Equation muſt be poſſeſſed by unknown, and the other by known Quantities, with the Sign of *Equality* between them; and thus will the unknown Quantity be diſcovered: And this is called *Tranſpoſition*. From hence will follow theſe *Axioms*, or ſelf-evident Principles, which I beg you would get by Heart, at leaſt ſo as to know their Uſe and Meaning.

AXIOM 1.

If equal Numbers or Quantities be added to equal Numbers or Quantities, their Sum will ſtill be equal; that is, ſuppoſe a was $= 4$, then by adding any Number or Quantity to each Side of the *Equation* (ſuppoſe 12) it will ſtill be equal; that is $a + 12 = 4 + 12$, $= 16$.

AXIOM 2.

If equal Quantities or Numbers be ſubtracted from equal Quantities or Numbers, the Remainder will be equal. Thus, ſuppoſe $x = 12$, then by ſubtracting 8 from each Side, $x - 8 = 12 - 8 = 4$.

AXIOM 3.

If equal Quantities be multiplied by equal Quantities, the Products will be ſtill equal. Thus, ſuppoſe $x = 8$, and I multiply each Side by any Quantity or Number, as 12, then will $12x = 96$. This is plain from the next *Axiom*.

AXIOM 4.

If equal Quantities be divided by equal Quantities, the Quotients will be equal. Suppoſe $12x = 96$, then dividing by 12, x will be equal to 8, as *per Axiom* 3.

AXIOM 5.

Thoſe Numbers or Quantities that are equal to one and the ſame Thing, are equal to one another; that is, ſuppoſe x, or $b - c$, or $5e + 6$, were either of them $=$ to 144, then are they alſo equal to each other.

You will ſee more of the Nature of theſe *Axioms* in the next *Section*, in treating of *Tranſpoſition*.

SECT. II.

REDUCTION *by* ADDITION, *or*, *the Method of* TRANSPOSING *Numbers and Quantities.*

Tyr. WHAT do you mean by *Tranſpoſition?*

Phi. *Tranſpoſition* is the *tranſpoſing*, *altering*, *changing*, or *removing* any Thing from one Place to another. To *tranſpoſe* then any Number or Quantity, is only to remove it from one Side of the *Equation* and placing it on the other with the contrary Sign; and this anſwers to *Axiom* the *1ſt* and *2d*.

Tyr. If I remember, you uſe this Character (φ) for *Tranſpoſition*: Do you not?

Phi. Yes, I ſhall throughout the Work, and where ever you meet with it read the Word *tranſpoſing*.

Tyr. Very well. Pleaſe to give me ſome Examples in *Addition?*

☞ *Phi.* I will; and pray remember, that *Addition* is nothing more than removing every *negative Quantity* to the contrary Side of the *Equation*, and making it *affirmative.*

EXAMPLE 1.

Suppoſe 1 $x - b = c$. Then $\varphi - b$
2 $x = c + b$ *Anſ.*

Or, by *Axiom* 1, adding $+ b$ to each Side of the *Equation*, it will be

1 $x - b = c$
2 $+ b = + b$
3 $x = c + b$ as before; becauſe $- b + b$ on the firſt Side of the *Equation* deſtroy each other.

EXAMPLE 2.

Let 1 $x - d - b = g$. Then $\varphi - b$
2 $x - d = g + b$. And $\varphi - d$
3 $x = g + d + b$. *Anſ.*

Or by adding $+ d + b$ to each Side,

1 $x - d - b = g$
2 $+ d + b = + d + b$, it will be
3 $x = g + d + b$ as before, becauſe $- d + d$, and $- b + b$, on the firſt Side, deſtroy each other.

Thus you ſee *Tranſpoſition* agrees with *Axiom* 1.

Tyr. I perceive it does; but it is leſs Trouble to change the Signs, than it is to add equal Quantities on each Side.

Phi. It is; but ſtill *Axiom* 1. ſhews you the Reaſon of it, which perhaps you might not have known elſe.

Tyr. Pleaſe to give me another Example, and prove it by Numbers?

Phi. Obſerve then.

EXAMPLE 3.

Let 1 $x - bc - d = aa$. Then $\varphi - d$
2 $x - bc = aa + d$. And $\varphi - bc$
3 $x = aa + d + bc$ *Anſ.*

Numerical Proof.

The Equaton is $x - bc - d = aa$. Make $-bc = -12$, $-d = -8$, and $aa = 25$. Then will it be

1 | $x - 12 - 8 = 25$. Then $\varphi - 8$.
2 | $x - 12 = 25 + 8$. And $\varphi - 12$.
3 | $x = 25 + 8 + 12 = 45 = aa + d + bc$ as before.

And thus you ſee that Quantities may be repreſented by any Numbers at Pleaſure, and the Value of the unknown Quantity x may eaſily be diſcovered.

Tyr. I like this very well indeed. Give me ſome more Examples.

Phi. I will.

EXAMPLE 4.

Let
1 | $5x - 8 = 24 - x$. Then $\varphi - 8$
2 | $5x = 24 + 8 - x$. And $\varphi - x$
3 | $5x + x$, that is, $6x = 24 + 8 = 32$ *Anſ.*

EXAMPLE 5.

Let
1 | $x - d - bc = aa - 12b$. Then $\varphi - bc$
2 | $x - d = aa - 12b + bc$. And $\varphi - d$.
3 | $x = aa - 12b + bc + d$. Laſtly, $\varphi - 12b$
4 | $x + 12 = aa + bc + d$. *Anſ.*

Do you underſtand it?

Tyr. Yes, very plainly.

Phi. Then we will proceed to *Subtraction*, in which I ſhall give you the ſame Sort of Examples as in Addition, that you may ſee the Nature of both the better.

SECT. III.

REDUCTION *by* SUBTRACTION.

Tyr. I Underſtand *Addition* very well, and apprehend *Subtraction* to be only the Reverſe of it.

Phi. You are right, for here you have Nothing to do but to *tranſpoſe* the *affirmative* Quantities or Numbers to the other Side of the *Equation*, and place the *negative* Sign before them.

EXAMPLE 1.

Let 1 | $x + b = c$. Then $\varphi + b$.
2 | $x = c - b$. *Anſ.* See *Ex.* 1. *Addition.*

Or, by *Axiom* 2, ſubtracting $+ b$ from each Side, it will be the ſame. Thus,

1 | $x + b = c$. Then ſubtracting
2 | $- b = - b$, it is
3 | $x = c - b$ as before; for $+ b - b$ deſtroy each other on the firſt Side.

EXAMPLE 2.

Let 1 | $x + d + b = g$. Then $\varphi + b$
2 | $x + d = g - b$. And $\varphi + d$
3 | $x = g - b - d$. *Anſ.*

Or by ſubtracting $+ d + b$ from each Side.

1 | $x + d + b = g$. Then
2 | $- d - b = - d - b$
3 | $x = g - b - d$, as above; becauſe $+ d + b - d - b$ on the firſt Side deſtroy each other.

EXAMPLE 3.

Let 1 | $x + bc + d = aa$. Then $\varphi + d$
2 | $x + bc = aa - d$. And $\varphi + bc$
3 | $x = aa - d - bc$. *Anſ.*

Numerical Proof.

The *Equation* is, $x + bc + d = aa$. Now let $+ bc = 12$, $d = 8$, and $aa = 25$. Then it will be

1 | $x + 12 + 8 = 25$. Then φ 8
2 | $x + 12 = 25 - 8$. And φ 12
3 | $x = 25 - 8 - 12 = 5$. *Anſ.* $= aa - d - bc$, as above.

Pleaſe to compare this with *Example* 3 in *Addition* you will ſee the Quantities are the ſame, but the Difference of the Value of x is 40 leſs here than it is there, becauſe you ſee that what is *affirmative* there is *negative* here. And indeed I am of Opinion, that the comparing of them with each other will ſhew you more the Nature of each, than many Examples whoſe Steps are not alike, and confuſedly demonſtrated.

Tyr. Indeed I think it almoſt impoſſible not to underſtand it, ſo plain as you have done it.

Phi. Here follows then

EXAMPLE 4.

Let 1 $5x + 8 = 24 + x$. Then $\varphi + 8$
2 $5x = 24 - 8 + x$
3 $5x - x$, that is, $4x = 24 - 8 = 16$.

See *Example* 4 in *Addition.*

EXAMPLE 5.

Let 1 $x + d + bc = aa + 12b$. Then $\varphi\ bc$
2 $x + d = aa + 12b - bc$. And $\varphi\ d$
3 $x = aa + 12b - bc - d$. Laſtly, $\varphi\ 12b$
4 $x - 12b = aa - bc - d$ *Anſ.* Compare this with *Example* 5 in *Addition.*

Tyr. I ſee the Nature of both plainly. Have you any Thing further to add?

Phi. It may not be amiſs to give you an Example to exerciſe you in both.

An EXAMPLE *in both* RULES.

EXAMPLE 5.

Let 1 $4x - g + b + c - d = 142$. By $\varphi - g$
2 $4x + b + c - d = 142 + g$. By $\varphi\ b$,
3 $4x + c - d = 142 + g - b$. Then $\varphi\ c$,
4 $4x - d = 142 + g - b - c$. And $\varphi - d$,
5 $4x = 142 + g - b - c + d$ *Anſ.*

Numerical Proof.

Let $-g = -8$, $+b = 12$, $+c = 6$, and $-d = -16$; what will x be?

Then,
1 $4x - 8 + 12 + 6 - 16 = 142$. By $\varphi - 8$,
2 $4x + 12 + 6 - 16 = 142 + 8$. By φ 12,
3 $4x + 6 - 16 = 142 + 8 - 12$. By φ 6,
4 $4x - 16 = 142 + 8 - 12 - 6$. And φ 16,
5 $4x = 142 + 8 - 12 - 6 + 16$ *Anſ.* as above. That is,
6 $4x = 166 - 18$. That is,
7 $4x = 148$. Therefore by dividing 148 by 4,
8 $x = \frac{148}{4} = 37$ *Anſ.*

Tyr. I like this *numerical* Proof very much, it is ſo plain, and the firſt five Steps agree ſo with the *literal*, that there need no more Examples of this Sort.

☞ *Phi.* I have proceeded indeed but one Step and one Quantity at a Time, becauſe you might ſee the gradual Order of the Work; but you may as well *tranſpoſe* them all at one Stroke, for it is only uſing the contrary Sign you know; however, this is left to your Liberty and Practice.

Tyr. I underſtand you very well. Pray what comes next?

Phi. You are now come to *Multiplication*, where you will begin to ſee the Beauty of *Equations*.

SECT. IV.

REDUCTION *by* MULTIPLICATION.

Tyr. HOW is *Multiplication* of *Equations* performed?

Phi. *Multiplication* is performed as follows:

1. When there is an *Equation* between two *Fractions* having a common Denominator, then caſt away or cancel the common Denominator, and the Numerators will be equal to each other.

EXAMPLE 1.

Let 1 $\frac{x}{9}=\frac{ab}{9}$. Then
2 $x=ab$. *Anſ.*

2. Or, if the *Fractions* have not a common Denominator, reduce them to one, after which expunge the common Denominator, and the new Numerators will be equal.

EXAMPLE 2.

Let 1 $\frac{x}{5}=\frac{6}{10}$. Theſe reduced to a C. D.
2 $\frac{10x}{50}=\frac{30}{50}$, that is
3 $10x=30$. *Anſ.*

3. Or, if there be but one *Fraction*, and that be made equal to any whole Number or Quantity, then only multiply the whole Quantity by the Denominator of the *Fraction*, and that Product ſhall be equal to the Numerator.

EXAMPLE 3.

Let 1 $x=\frac{24a}{12}$ then
$12x=24a$ *Anſ.*

4. Or, to prevent the Trouble of reducing the *Fractions* to a common Denominator, multiply the Numerator of the ſecond *Fraction* by the Denominator of the firſt *Fraction*, and place the Product for a new Numerator over the ſecond *Fraction*, ſo will the Numerator of the firſt *Fraction* be equal to it in the ſecond Step. Then multiply the Numerator of the firſt *Fraction* by the Denominator of the ſecond *Fraction*, ſo will this Product be equal to the ſaid new Numerator, and the *Equation* will be cleared from *Fractions*, and the unknown Quantity diſcovered.

EXAM-

EXAMPLE 4.

Let 1 $\frac{x}{12}=\frac{108}{4}$. Then 108 × 12
2 $x=\frac{1296}{4}$. Then $x \times 4$
3 $4x=1296$. Then muſt
4 $x=\frac{1296}{4}=324$ *Anſ.*

Tyr. I like this Way beſt, I muſt confeſs.

Phi. When the Numbers or Quantities are many, it is leſs Trouble indeed to do it this Way. Now perhaps you will be diverted with the Proof.

Tyr. I ſhould be glad to ſee the Reaſon indeed why $\frac{x}{12}$ is equal to $\frac{108}{4}$.

Phi. That you ſhall directly, and in a few Words. Firſt, x is proved to be equal to 324, as above; then $\frac{324}{12}=\frac{108}{4}=27$. So that $\frac{x}{12}=\frac{108}{4}$ Q. E. D.

Tyr. Very pretty indeed! But pray give me Leave to ſet you a Queſtion, and to deſire you to do the Work at large: I aſſure you it will be of great Service to me, and I ſhall need no more Examples of this Sort?

Phi. You know or may know, I am always ready to ſerve you. Pray propoſe the Thing?

EXAMPLE 5.

Tyr. *Suppoſe* $\frac{3x}{4}+12=\frac{2x}{3}+14$, *what then is* x *equal to?*

Phi. A very pretty uſeful Queſtion, and I will do it ſo plain that I believe you will be ſatisfied with the Manner of it, becauſe I ſhall demonſtrate it as a ſtanding Rule for all ſuch Examples. Let $\frac{3x}{4}+12$ $=\frac{2x}{3}+14$.

DEMON-

DEMONSTRATION

Firſt, I multiply the whole Number 12, and the Numerator of the ſecond *Fraction*, (*viz.* $\frac{2x}{3}$) and the whole Number 14 into the Denominator of the firſt *Fraction*, *viz.* 4, and the Products are 48, $8x$ and 56: But whereas the Sign of Equality ($=$) falls between the whole Number 12 and the *Fraction* $\frac{2x}{3}$, I ſtill keep it always in the ſame Place, till I have done multiplying the whole, therefore it will be in the ſecond Step, thus, $3x + 48 = \frac{8x}{3} + 56$. Then to clear the ſecond Side from *Fractions*, I now multiply every Member of the ſecond Step into the Denominator of the ſecond *Fraction*, *viz.* 3, except it be its own Numerator $8x$, and then it will be in the third Step $9x + 144 = 8x + 168$, and thus is the whole *Equation* freed from *Fractions*. Now tranſpoſing $8x$ and $+ 144$, I have in the fourth Step $9x - 8x = 168 - 144$; that is, $x = 168 - 144 = 24$, the Value of x required.. (See the Work at large as follows, and compare it with what is above.)

The Operation of EXAMPLE 5.

Let | 1 | $\frac{3x}{4} + 12 = \frac{2x}{3} + 14$. Then × 4
| 2 | $3x + 48 = \frac{8x}{3} + 56$. Which × 3, the Denominator of the ſecond *Fraction* is
| 3 | $9x + 144 = 8x + 168$. Then φ $8x + 144$,
| 4 | $9x - 8x = 168 - 144$. That is,
| 5 | $x = 24$. *Anſ.*

PROOF 2.

To prove that $\frac{3x}{4} + 12 = \frac{2x}{3} + 14$.

Firſt $x = 24$ as above, then muſt $\frac{3x}{4} + 12$, that is, $\frac{72}{4} + 12 = 30$; and ſo alſo $\frac{2x}{3}$ that is $\frac{48}{3} + 14 = 30$. Q. E. D.*

Tyr. Nothing can be plainer, nor more eaſy to be underſtood.

Phi. If due Attention be given, as you obſerve, it is eaſy enough; I ſhall therefore leave one more Example with you, and haſten to *Diviſion*

EXAMPLE 6.

Let
1 $\frac{4x}{9} + 16 = \frac{3x}{6} - 18$. Firſt × 9
2 $4x + 144 = \frac{27x}{6} - 162$. This × 6,
3 $24x + 864 = 27x - 972$. Then φ $24x$ and -972,
4 $864 + 972 = 27x - 24x$. That is,
5 $1836 = 3x$. Therefore
6 $x = \frac{1836}{3} = 612$ *Anſ.*

Again.

Suppoſe that $36 + \frac{6x}{8} = 72$, then will x be found to be $= 48$.

☞ But here, *Tyrunculus*, you muſt obſerve, that if at any Time the *Square* or *Cube* of any unknown Quantity ſhould in the laſt Step of any *Equation*, be found to be equal to ſuch Number or Quantity, then muſt you extract the *Square* or *Cube Root* of ſuch Numbers, and you will then have the Value of the unknown Quantity itſelf. Thus, ſuppoſe xx ſhould at laſt fall out to be equal to 81, then is $x = 9$ the *Square Root* thereof; and if $xxx = 64$, then will $x = 4$, its *Cube Root*.

---

* *N. B.* This is called *Synthetical Demonſtration*, or *Compoſition*, and you may ſee this *Equation* turned into a *Problem*, and ſolved *Algebraically*, *Dialogue* 10, *Problem* 29.

EXAMPLE 7.

Let
1 $\frac{xx}{4} = 44 + 5$. Then
2 $xx = 176 + 20$, that is,
3 $xx = 196$. Therefore,
4 $x = \sqrt{196} = 14$ *Anſ.*

EXAMPLE 8.

Let
1 $\frac{xxx}{b} + c + f = dg$
Then, $\times b$
2 $xxx + bc + bf = bdg$
3 $x = \sqrt[3]{bdg - bc - bf}$.

Tyr. I need no more Examples, what you have ſhewn me already is ſufficient.

Phi. You may indeed reduce any ſimple *Equation* by what you have ſeen, therefore we will proceed to *Diviſion.*

SECT. V.

REDUCTION *by* DIVISION.

Tyr. PRAY how is *Diviſion* of *Equations* performed?

Phi. When any Quantity or Quantities that are alike, poſſeſs both Sides of the *Equation*, divide each Side by the ſaid Quantity, (which is the ſame as to reduce it to its loweſt Terms) and then will one Side be ſtill equal to the other: and if there be *Fractions*, clear the *Equation* of them, by multiplying all the Parts by the Denominators of the *Fraction*, as in *Multiplication*.

EXAMPLE I.

Let
1 $xx = 16x + 12x$, then $\div$ by x,
2 $x = 16 + 12 = 28$, by *Axiom* 4.

EXAM-

EXAMPLE 2.

Let
1 | $xxc + bcx + dcx = cx + cffx$. Then $\div c$
2 | $xx + bx + dx = x + ffx$. Then $\div x$,
3 | $x + b + d = 1 + ff$ *Anſ.*

EXAMPLE 3.

Let
1 | $bx - cx = gg$. Then $\div \overline{b - c}$,
2 | $x = \frac{gg}{b-c}$ *Anſ.*

Tyr. I do not rightly apprehend this laſt Example. Can you demonſtrate it by Numbers?

Phi. Yes to be ſure, and will. Let $b = 8$, $-c = -4$, and $g = 12$, then $gg = 144$.

Numerical Proof of EXAMPLE 3.

Let
1 | $8x - 4x = 144$, then $\div$ by $8 - 4$
2 | $x = \frac{144}{8-4} = \frac{gg}{b-c} = 36$ *Anſ.*

Tyr. I heartily thank you, *Philomathes*, and am mightily pleaſed with it. But if there be *Fractions*, what do you ſay I am to do as I did in *Multiplication?*

Phi. Yes, after having firſt abbreviated the Numerators, or divided them by like Quantities or Numbers, (for the Denominators are never divided) you proceed then to multiply croſs-ways, as in *Diviſion* of *Fractions*, till you diſcover the unknown Quantity.

Tyr. Pleaſe to give me an Example?

Phi. I will.

EXAMPLE 4.

Let 1 $\frac{84x}{x-4}=\frac{70x}{x-6}$. Then ÷ Numerators by x, you have

2 $\frac{84}{x-4}=\frac{70}{x-6}$. Then $\overline{x-4}\times 70$

3 $84=\frac{70x-280}{x-6}$. Then $84\times\overline{x-6}$,

4 $84x-504=70x-280$. Then φ $70x$ and -504,

5 $84x-70x=504-280=224$; that is,

6 $14x=224$. Therefore,

7 $x=\frac{224}{14}=16$ *Anſ.*

Tyr. And can you prove this laſt Example *ſynthetically*, as you did in *Multiplication?*

Phi. Yes, moſt certainly, if the Work be done right.

A ſynthetical Proof of EXAMPLE 4.

You ſee that x is $=16$, then $\frac{84x,}{x-4}$ that is, $\frac{84\times 16}{16-4}=112$, the firſt Side of the *Equation.* Again, $\frac{70x}{x-6}$ that is, $\frac{70\times 16}{16-6}=112$. Conſequently therefore, $\frac{84x}{x-4}=\frac{70x}{x-6}$ Q. E. D.

Tyr. Not one Thing that you have demonſtrated pleaſes me better, nor gives me greater Satisfaction.

Phi. I will give you an Example or two more.

EXAMPLE 5.

Let 1 $ddxx+ddbxx-ddx=ddfx+ddffx$. Then ÷ dd

2 $xx+bxx-x=fx+ffx$. Then ÷ x

3 $x+bx-1=f+ff$.*

---

* From $x+bx-1=f+ff$, we get, by φ 1,
$x+bx=1+f+ff$; and ÷ $\overline{1+b}$,
$x=\frac{1+f+ff}{1+b}$. For $x\times\overline{1+b}=x+bx$.
EDIT.

☞ Here indeed the known Quantities and the unknown Quantity (which we ſuppoſe is repreſented by x) both poſſeſs the firſt Side of the *Equation*; in order therefore to let the unknown Quantity x poſſeſs one Side by itſelf, do thus: Let the whole *Equation* be divided by the known Quantity or Quantities, and then will the unknown Quantity x be equal to the Quotient of ſuch Diviſion. As

EXAMPLE 6.

Suppoſe in any *Equation* it ſhould ſo fall out, that $xd - xb = c + g$, what is x equal to?

Let | 1 | $xd - xb = c + g$. Then $\div \overline{d - b}$
| | $x = \frac{c +}{d - b}$ *Anſ.*

EXAMPLE 7.

☞ Again, Suppoſe in trying to diſcover the unknown Quantity, all the Quantities happen to fall together, ſo as there is no Sign of *Equality* between; then, in order to form an *Equation*, make ſuch Quantities the firſt Side, and put a Cypher on the ſecond Side of the *Equation*, ſo will the unknown Quantity be diſcovered. Thus,

Suppoſe | 1 | $12x - 312$, then
| 2 | $12x - 312 = 0$; that is,
| 3 | $12x = 312$. Therefore,
| 4 | $x = \frac{313}{12} = 26$

N. B. See the 9th and 10th Steps of *Problems* 18. *Dialogue* 10.

EXAMPLE 8.

Let | 1 | $5xxd + 8xx = xbc + 4xb$, then $\div\ x$
| 2 | $5xd + 8x = bc + 4b$, then $\div\ 5d + 8$
| 3 | $x = \frac{bc + 4b}{5d + 8}$. *Anſ.*

SECT.

SECT. VI.

How to convert or turn EQUATIONS *into* ANALOGIES, *and the contrary.*

Tyr. I Imagine that this *Section* depends upon a true Knowledge of the Nature of *Proportion*; does it not?

Phi. Moſt certainly, and therefore from what has been laid down in *Dial.* 7. *Sect.* 2. it will be eaſy to convert any *Equation* into an *Analogy*, or right *Proportion*; and eſpecially ſince I ſhall take ſome of the ſame *Equations*, and refer you back to the former Work, to confirm you the better in what you are doing.

OBSERVATION I.

When any *Equation* (not having *Fractions*) is given to be converted into an Analogy, then it will be, as any of the *Quantities* or *Factors* on one Side are to any *other* on the other Side; ſo will the remaining *Quantity* or *Quantities* on the ſame Side, be to the remaining *ones* on the other Side, and *vice verſa*.

EXAMPLE I.

Let the *Equation* be $xd = cb$. *See the third Step in the Demonſtration.* Dial. 7. Sect. 2.

Let
1 | $xd = cb$, then
2 | As $x : c :: b : d$. Or
3 | As $x : b :: c : d$, &c. For the 1 Term
4 | $x \times$ (the 4) $d = (2)\ c \times (3)\ b$. Conſequently
5 | $xd = cb$.

Tyr. I perceive then, this is but a common Proof to *Proportion*.

Phi. Nothing more; for if you compare this *Example* with the ſix *Obſervations* laid down in *Dial.* 7. *Sect.* 2. you may make a great many more Steps of it than I have done.

Tyr. I ſee plainly the Manner of turning *Equations* into *Analogies* when both Sides are whole Quantities; but ſuppoſe one Side be a *whole Number* or *Quantity*, and the other a *Fraction?*

Phi. Then you are to proceed as follows.

OBSERVATION 2.

When any *whole Number*, or Quantity in an *Equation* is made equal to a *Fraction*, whoſe Numerator has two Quantities and the Denominator but one; then break the Numerator into two ſuch Parts, which multiplied together, will produce the ſame, and make thoſe Parts the *Means*; then make the whole Quantity and the Denominator of the *Fraction* the *Extremes*. Or in other Words, make the whole Quantity the firſt Term in the *Rule of Three*, the Denominator of *Fraction* the fourth, and the Numerator divided into two Parts, as before directed, make the ſecond and third Term.

EXAMPLE 2.

Let 1 $x = \frac{bc}{d}.$ Then
2 $x : b :: c : d.$ Or
3 $x : c :: b : d.$ That is,
4 $xd = bc.$ See *Demonſtration*, *Dial.* 7. *Sect.* 2.

Tyr. I underſtand it very well, but ſuppoſe both be *Fractions*, how then?

Phi. Certainly you forget, *Tyrunculus*. Pray turn back to the fourth Step of *Dialogue* 7. *Sect.* 2. for I ſhall give you the ſame *Example*. Or if you remember what I told you in *Abbreviations*, you will find the *Analogy* will hold as follows.

OBSERVATION 3.

As one Denominator is to the other, ſo will one Numerator be to the other; or as one Denominator is to its own Numerator, ſo is the other Denominator to its Numerator.

EXAMPLE 3.

Let
1 $\frac{b}{x} = \frac{d}{c}$. Then as
2 $x : c :: b : d$. Or as
3 $x : b :: c : d$, *&c. &c.*

Tyr. I am obliged to you: Have you nothing more to add?

Phi. I will give you an *Example* by Way of Exerciſe.

EXAMPLE 4.

Let
1 $xb + xd = bd$. Then as
2 $x : b :: d : b + d$. For
3 $x \times \overline{b + d} = xb + xd$, the firſt Side. And
4 $b \times d = bd$, the ſecond Side. Or as
5 $b : x :: b + d : d$. Or, by adding xd to each Side,
6 $xb + 2xd = bd + xd$. Then, as
7 $x : b + x :: d : b + 2d$. Or, taking xd from each Side,
8 $xb = bd - xd$. Then, as
9 $x : d :: b - x : b$, *&c. &c.* conſequently
10 $xb + xd = bd$.

Tyr. Then I perceive by the third and fourth Steps that if one Side of an *Equation* can be divided into two Parts, ſo as to become *Extremes* (which being multiplied together, will be equal to the Side before it was divided) the other Side being divided in the ſame Manner, will be the *Means*, will it not?

Phi. Yes, your Notion is right; and I am glad to find you ſo perfect in what you have done: Therefore I ſhall bid you *adieu*, and leave you to conſider upon thoſe *Examples*, which you think yourſelf leaſt acquainted with; and when Opportunity ſuits, I ſhall be glad to ſee you and your Friend *Novitius*, and then we will put theſe *Examples* in Practice by ſome *Algebraic Problems*.

Tyr. Sir, I am obliged to you; and I dare ſay *Novitius* will be as proud of the *Invitation*.—But let me beg of you to ſtay a little longer.

Phi. Not now, *Tyrunculus*, I think I have made you a long Viſit; beſides Night comes on a-pace, and I chooſe to go.

Tyr. Sir, if you are determined to go, I heartily wiſh you a good Night, and humbly thank you for your Company, and I intend to do myſelf the Pleaſure of waiting upon you very ſhortly.

Phi. When you think proper, *Tyrunculus*.

CHAP. III.

DIALOGUE IX.

SECT. I.

Between PHILOMATHES *and* TYRUNCULUS, *concerning the Nature of* Algebraic Problems, *and how to prepare them for a* Solution.

Tyrunculus *returning the Viſit to* Philomathes.

Tyr. P*HILOMATHES*, you humble Servant, how do you do?

Phi. Thank you, *Tyrunculus*, I am pretty well, and am glad to ſee you ſo.

Tyr. You remember I ſaid it ſhould not be long before I would call again to ſee you; but, perhaps, I am not come at a ſuitable Time.

Phi. You could not have hit upon it better, *Tyrunculus*; it ſuits me quite well; and I was but juſt before thinking of, and wiſhing for you.—Come, pray ſit down. —But where is your Friend *Novitius*, I expected you both together?

Tyr. We are obliged to you, Sir, and I aſked him to come, as you deſired, and he promiſed to follow me.

Phi. Well, *Tyrunculus*, in the mean Time let me know how you go on, and what Improvement you have made ſince I ſaw you laſt.

Tyr. I am afraid it will not bear too cloſe an Examination: However, that very Night you left me, I looked over the chief of what you have ſhewn me, and find myſelf much more perfect in it.

Phi. You have done well, it is all I required, and you will be the better able to underſtand the following *Problems.*

Tyr. I muſt confeſs I do not care how ſoon I begin to try a few Queſtions, or at leaſt ſee them wrought, for you muſt know I am in a Hurry.

Phi. You ſhall preſently; but pray be not ſo over haſty: *fair* and *ſoftly*, you know, go the furtheſt; and I have ſomething to premiſe firſt of all, that will be of Service, and help you forward in the Work.

Tyr. Pray what is that?

Phi. It would be requiſite that you ſhould be acquainted with the following Obſervations.

OBSERVATION I.

When any Queſtion is given to be anſwered in an *Algebraic* Manner, *firſt*, For the Anſwer or Number ſought, put x: Then proceed according to the Tenor of the Queſtion, to *add*, *ſubtract*, *multiply*, or *divide*, until you have formed an *Equation*, which if it has *Fractions* muſt be cleared according to the Rules laid down in *Multiplication*; *Sect.* 4. *Dial.* 8: This done, proceed to *tranſpoſe* according to the Order of *Addition* and *Subtraction* of *Equations*, and you will (by keeping x on the firſt Side of the *Equation*) have it equal at laſt to ſome known *Quantity* or *Qualities*, by which alſo x will be of Courſe known, and its Value diſcovered.

OBSERVATION 2.

Though it be cuſtomary to uſe x for the unknown Quantity, yet you may make Uſe of any other Letter at Pleaſure. Some *Analiſts* uſe Vowels to repreſent *unknown*, and Conſonants *known* Quantities; but others uſe them as their own Fancy and Inclination direct: But ſtill you are to obſerve, the Letter (*o*) is never uſed to expreſs a Quantity, (though indeed the Anſwer would be the ſame with this as with any other Letter,) and there ſeems to be a Reaſon for it, ſince it is but a Cypher at beſt without Integers; and therefore, ſince Nothing cannot be Something, by Reaſon of its Want or Deficiency, it would be abſurd to put it to repreſent any Number or Quantity; though, as I obſerved before, it is ſometimes uſed to form an *Equation*. See *Problem* 18. *Step*. 10.

OBSERVATION 3.

If to the Sum of any two Numbers you add their Difference, and divide the Whole by 2, the Quotient will be the greater Number. Or if you add the Numbers and their Difference together, and divide by 2, you have the greater Number.

OBSERVATION 4.

If from the greater Number you take the Difference of the ſaid Numbers, the Remainder will be the leſs Number. Or, if you add any two Numbers and their Difference together, and divide the Sum by 2, and then ſubtract the Difference of the ſaid Numbers from the Quotient, the Remainder will be the leſs Number.

☞ OBSERVATION 5.

When any *Fraction* is given to be divided into two, three, or more Parts, then divide the Numerator, if you can, into ſuch Parts, and let the Denominator remain as it was; and in Caſe you cannot divide the Numerator into the Parts required, multiply the Denominator into ſuch Parts as are required, and let the Numerator remain as it was; ſo is the *Fraction* truly divided into ſuch Parts as really as if it had been performed by *Diviſion*, which is ſometimes very difficult. Thus, ſuppoſe I was to divide $\frac{6}{x}$ into three Parts, I divide 6 by 3, and it is 2; ſo is $\frac{2}{x}$ the $\frac{1}{3}$ of $\frac{6}{x}$. But ſuppoſe it were $\frac{x}{6}$ to be divided into 3 Parts, as I cannot well divide x by 3, therefore I multiply the Denominator 6 by 3, and it is 18; ſo is $\frac{x}{18}$ $\frac{1}{3}$ of $\frac{x}{6}$. This I have demonſtrated, becauſe you ſhall ſeldom meet with it in any Authors, although it is of infinite Service in *Algebra*. I beg therefore you would remember it in particular.

OBSERVATION 6.

When any two Numbers are given, and you would expreſs them *literally*, (we will ſuppoſe you put x for the greater, and e for the leſs Number) then will the following Steps be of Service, becauſe they will help you to underſtand the Nature of a Queſtion, and the ſooner to do the ſame, as being a proper and neceſſary Exerciſe of the foregoing Rules, by teaching you how to expreſs them in their true Order. Thus,

1 Suppoſe the greater be x,
2 And the leſs Number e,
3 Then will their Sum be $x + e$,
4 Their Difference $x - e$,
5 Their Product $x \times e$, *viz.* xe,
6 The Quotient of the greater ÷ by the leſs is $\frac{x}{e}$,
7 The Quotient of the leſs by the greater is $\frac{e}{x}$,
8 The Order of Proportion, as $x : e :: e : \frac{ee}{x}$.
9 Or, by putting the leſs firſt, as $e : x :: x : \frac{xx}{e}$.
10 The Square of the greater xx,
11 The Square of the leſs ee,
12 The Sum of their Squares $xx + ee$,
13 The Difference of their Squares $xx - ee$,
14 The Sum of their Sum and Difference $2x$,
15 The Difference of their Sum and Difference $2e$,
16 The Product of their Sum and Difference $xx - ee$,
17 The Square of their Sum $xx + 2xe + ee$,
18 The Square of their Difference $xx - 2xe + ee$,
19 The Square of their Product $xxee$,
20 The Cube of the greater xxx, or x^3,
21 The Cube of the leſs eee, or e^3, &c.

Theſe being underſtood, you may proceed to the working of the following *Algebraic Problems*.

DIALOGUE X.

SECT. I.

ALGEBRAIC PROBLEMS, *or the Solution of Queſtions producing* SIMPLE EQUATIONS.

Between PHILOMATHES, TYRUNCULUS, *and* NOVITIUS; *being a proper Exerciſe of all the foregoing Rules.*

Tyr. THIS is *that* Part of *Algebra* that I have ſo long wiſhed to be trying at, and to which by your kind Aſſiſtance, *Philomathes*, I am at laſt happily arrived.

Phi. I am as much ſatisfied, and take as great Pleaſure in your Progreſs as you poſſibly can, and I doubt not of your underſtanding the Manner of working the *Problems* in a ſhort Time. Only take Care to mind the Steps in the *numerical* Work, and you will ſoon underſtand the *literal*, for I ſhall endeavour to make the Steps alike if I can. And though you be perfect in the Chief of what you have done, yet give me Leave once more to remind you of theſe three Things, *viz.* That this Character (φ) in any Step ſhews you that the Number or Quantity before which it is placed is tranſpoſed in the next Step to the other Side of the *Equation*; this (Q.) ſignifies *by the Queſtion*; and laſtly, to remember that to take the $\frac{1}{2}$ or $\frac{1}{3}$, *&c.* of any *Fraction* is only to multiply the Denominator by 2, 3, *&c.* which is the ſame as to divide the Numerator by the ſame Figures. Theſe being obſerved, we will proceed to

PROBLEM I.

What Number is that which being multiplied by 12, and having 18 added to the Product, the Sum will be 294?

Numerical Solution.

Put 1 | x for the Number, this $\times$ 12
2 | $12x$; add 18, it is
3 | $12x + 18$. This $Q = 294$. Then φ 18
4 | $12x = 294 - 18$; that is,
5 | $12x = 276$. Then is,
6 | $x = \frac{276}{12} = 23$ *Anſ.*

Literal Solution.

Let $b = 12$, $c = 18$, $d = 294$.
1 | x as before $\times\ b$.
2 | xb; add $+\ c$
3 | $xb + c$. This $Q = d$
4 | $xb + c = d$. Then $\varphi\ c$.
5 | $xb = d - c$
6 | $x = \frac{d - c}{b} = 23$ *Anſ.*

PROBLEM II.

What Number is that, to which if I add 24, then from that Sum ſubtract 8, and multiply the Remainder by 5, the Product will be 320?

Numerical Solution.

Put 1 | x for the Number, then
2 | $x + 24$, then $- 8$,
3 | $x + 24 - 8$; this $\times$ 5, is
4 | $5x + 120 - 40$. Then Q.
5 | $5x + 120 - 40 = 320$. By φ 40
6 | $5x + 120 = 320 + 40$. Then φ 120
7 | $5x = 360 - 120 = 240$. Therefore
8 | $x = \frac{240}{5} = 48$ *Anſ.*

Literal Solution.

Let $b = 24$, $c = 8$, $d = 5$, $f = 320$.

Put
1 x as before
2 $x + b$
3 $x + b - c$. This $\times\, d$
4 $dx + db - dc$. Whence Q.
5 $dx + db - dc = f$.
6 $dx + db = f + dc$
7 $dx = f + dc - bd$
8 $x = \dfrac{f + dc - bd}{d}$* $= 48$

☞ But it is to be noted that all ſuch like Queſtions as *this* may be performed both ſhorter and eaſier, by working only with the Difference of the Numbers, and not the Numbers themſelves. Thus, you are deſired in the *Problem* to add 24, then ſubtract 8. Now it is evident that $24 - 8 = 16$; therefore if you work only with 16, by adding it to the unknown Quantity, it muſt be the ſame as to add 24 and ſubtract 8. So that the *Problem* may be read thus:

PROBLEM II. *in other Words.*

What Number is that, to which if I add 16, *and multiply that Sum by* 5, *the Product will be* 320?

Numerical Solution.

Put
1 x as before, then $+ 16$,
2 $x + 16$. This $\times\, 5$
3 $5x + 80$. Whence Q.
4 $5x + 80 = 320$. Then φ 80
5 $5x = 320 - 80 = 240$. Then
6 $x = \dfrac{240}{5} = 48$, as before.

---

* It is evident that $\dfrac{f + dc - bd}{d}$ is $= c - b + \dfrac{f}{d}$. EDIT.

Literal Solution.

Let $b = 16, f = 320, a = 5$

1 | x
2 | $x + b$. This × d
3 | $xd + bd$ Q.
4 | $xd + bd = f$
5 | $xd = f - bd$
6 | $x = \dfrac{f - bd}{d} = 48$, as before.

Tyr. This is much ſhorter and better indeed as you obſerve, and I begin to underſtand ſomething of the *literal* Operation, as well as the *numerical*; but I muſt needs ſay, I like the *numerical* beſt, I think it is the plaineſt for Learners.

Phi. Moſt are apt to ſay ſo indeed; but when once the other Way is known, you will like that as well, and to be ſure it is the ſhorter of the two, but I will not ſay the eaſier. However, I will perform all the *Problems* numerically, and ſome of them literally; and pray let me adviſe you to read over every Queſtion, at leaſt twice or thrice, that you may underſtand the Nature of it the better; for when once you have a true and juſt *Idea*, of the Intent and Meaning of it, you may be ſaid to have half done it.

PROBLEM III.

Says Alexander *to* Epheſtion, *I am older than you by two Years.* Clitus, *hearing it, ſaid, I know I am older than both of you by four Years. The Philoſopher,* Caliſthenes, *being preſent, ſaid, I remember I have heard my Father, who is now ninety-ſix Years, ſay, that he is as old as you all; I demand then the Ages of* Alexander, Clitus, *and* Epheſtion.

Numerical Solution.

Put
1 | x for *Epheſtion*'s Age; then will
2 | $x + 2$ be *Alexander*'s. Then + 4
3 | $2x + 6$ *Clitus*. Theſe added,
4 | $4x + 8$ their Sum; whence Q.

5 | $4x + 8 = 96$. Then φ 8
6 | $4x = 96 - 8 = 88$. Therefore
7 | $x = \frac{88}{4} = 22$ *Epheſtion*'s Age. And then
8 | $x + 2$, *viz.* $22 + 2 = 24$ *Alexander*'s
9 | $2x + 6 = 50$ *Clitus*'s.

Literal Solution.

Let $b = 2$, $d = 4$, $f = 96$
1 | x for *Epheſtion*'s Age
2 | $x + b$ *Alexander*'s,
3 | $2x + b + d$ *Clitus*'s,
4 | $4x + 2b + d$ Sum. Then Q.
5 | $4x + 2b + d = f$. Then φ $2b + d$
6 | $4x = f - 2b - d$
7 | $x = \frac{f - 2b - d}{4} = 22$ *Epheſtion*'s
8 | $x + b = 24$ *Alexander*'s
9 | $2x + b + d = 50$ *Clitus*'s.

Tyr. Mighty pretty; but why do you begin with *Epheſtion* rather than *Alexander?*

Phi. It would have been the ſame had I began with *Alexander*'s Age; only then the 2*d* Step would have been $x - 2$ for *Epheſtion*, and the 3*d* Step $2x - 2 + 4$; and ſo it would have occaſioned more Work, but now the Signs are all *affirmative*.

Tyr. I am ſatisfied, and begin to ſee a little more into it.

Phi. There is no fear of your underſtanding it, if you mind.

PROBLEM IV.

Three Perſons, A, B, *and* C, *trade and gain* 3000*l.* *the Share of* A *is to be but Half the Share of* B; *and the Share of* B *one Third the Share of* C; *I demand each Man's Share?*

Now to avoid *Fractions*, I begin with *A* firſt; for if I put x for *B*, then *A* muſt be $\frac{x}{2}$ and *C* $3x$. Therefore

Numerical

Numerical Solution.

Put 1 x for *A*'s ſhare; then is
2 $2x$ *B*'s Share, and
3 $6x$ *C*'s Share; theſe added, make
4 $9x$, the Sum. Whence Q.
5 $9x = 3000l.$ Therefore
6 $x = £.\,\frac{3000}{9} = 333l.\ 6s.\ 8d.$ *A*'s Share
7 And by the 2*d* Step 666*l.* 13*s.* 4*d.* *B*'s
8 And by the 3*d* 2000 *C*'s
9 Their Sum $= 3000$.

Literal Solution.

As x is put for *A*'s Share, and the other is double and treble, the firſt four Steps will be the ſame as the *numerical*. Now let
$b = £.\,3000$
5 $9x = b$
6 $x = \frac{b}{9} = £.\,333\frac{1}{3}$
7 $2x = 666\frac{2}{3}$
8 $6x = 2000$
9 Sum $= £.\,3000$ as before.

Do you underſtand theſe Operations?

Tyr. The *literal* Part I am not at preſent ſo much acquainted with, but the *numerical* appears quite plain and eaſy to me; I heartily wiſh *Novitius* was here, he would be ſo pleaſed to ſee ſome of thoſe Queſtions which puzzle him demonſtrated in ſo eaſy a Manner.

Phi. ———There is a young Gentleman now coming up the Walk.

Tyr. Perhaps it is he——It is ſo—I will go to the Door, for I know he is quite baſhful——

Phi. Stay——give me Leave, *Tyrunculus*; it will look better in me, and he will take it kinder at my Hands——

Nov. Your humble Servant, *Philomathes:* Pray is *Tyrunculus* here?

Phi. He is, pray come in, *Novitius.*

Nov. Sir,——Friend *Tyrunculus*, how fare you?

Tyr. I am a little vexed with you for ſtaying.

Nov. I aſk *Philomathes*'s Pardon in particular—I was unexpectedly prevented by an Acquaintance.

Phi. Well, *Novitius*, we will not uſe ſuperfluous Ceremonies at this Time: Pray ſit down, I am glad to ſee you.

Nov. Sir, I thank you.

Phi. Your Friend *Tyrunculus* was ſaying you had ſome Queſtions to aſk me that had puzzled you pretty much, pray, what are they?

Nov. Only ſome few of Mr. *Cocker*'s and *De-Billy*'s, for ſeveral of them are ſo contracted in the Work that I cannot underſtand them.

Phi. To do Juſtice to the firſt Author, I know not a prettier Piece for Learners on the firſt four Rules of *Algebra*; but I confeſs he is a little dark in ſome of the Operations. Come, I have him by me, and pray do you look out thoſe Queſtions that puzzle you moſt, and we will work them more plainly to your Underſtanding.

Nov. Pleaſe then to begin with his fifth Queſtion.

Phi. There are *Fractions* concerned in that; therefore I think we had better begin with the more eaſy ones firſt, and take the harder as they come in Courſe.

Nov. It is true; do ſo if you pleaſe.

Phi. Obſerve then.

PRO-

* PROBLEM V.—*Cocker's* 8*th* Queſtion.

A Labourer received 2l. 8s. for threſhing 60 Quarters of Corn, viz. *Wheat and Barley; for the Wheat he received 12 Pence a Quarter, and for the Barley 6 Pence: How many Quarters did he threſh of each.*

Numerical Solution.

1 For the Quarters of Wheat put x,

2 Then as both together are but 60 Quarters, the Barley muſt be $60-x$.

3 Now x Quarters of Wheat, at 12*d.* a Quarter, is $12x$.

4 And $60-x$ Quarters of Barley, at 6*d.* is $360-6x$.

5 Theſe two are equal Q. to 48*s.* or 576 Pence; whence $12x+360-6x=576$.

6 Then φ 360, $+12x-6x=576-360$

7 That is, $6x=576-360=216$.

8 Therefore $x=\frac{216}{6}=36$ Quarters of Wheat.

9 And by 2*d* Step $60-x$, or $60-36=24$ the Barley.

Literal Solution.

Let $b=576$ Pence, $c=60$, $d=12$, $f=6$ Pence.

1 x Wheat

2 $c-x$ Barley. Then $x\times d$ is

3 dx for the Wheat, and $f\times\overline{c-x}$ is

4 $fc-fx$ for the Barley. Whence Q.

5 $dx+fc-fx=b$, or 576 Pence. Then

6 $dx-fx=b-fc$. Therefore

7 $x=\frac{b-fc}{d-f}$; that is, $x=\frac{576-360}{12-6}=36$, as above, Wheat;

8 And $c-x$, or $60-36=24$, the Barley.

---

* Thoſe *Problems* that have an Aſteriſk before them, are inſerted by the Deſire of ſeveral young *Algebraiſts*, who wanted a plainer and eaſier Demonſtration than in the Original: And to ſuch as are unacquainted with either of the Authors, they will be equally ſerviceable, as if they were new ones.

PROOF.

36 Quarters, at 1*s*. = 36*s*.
24 Quarters, at 6*d*. = 12*s*.

48*s*.

Nov. I underſtand it quite well.

Tyr. So do I both the Ways.

Phi. I ſhall not write againſt the literal any more, but leave you to compare it with the *numerical Work*.

* PROBLEM VI.—*Cocker*'s 10*th* Queſtion.

A Gentleman hired a Servant for 40 *Days, upon this Condition, that every Day he wrought he was to receive* 20 *Pence, and for every Day he was idle, and did no Work, he was to pay* 8 *Pence. Now at the End of the Time he received* 15*s*. 4*d*. *How many Days did he work, and how many was he idle?*

Numerical Solution.

1 For the Days he wrought put x.
2 Then will what he played be $40-x$.
3 Now x Days wrought, at 20*d*. a Day, is $20x$.
4 And $40-x$ play'd at 8*d*. a Day, is $320-8x$
5 Subtract the 4*th* Step from the 3*d* Step, it is $20x-320+8x$
6 This being his Due is (Q.) = 15*s*. 4*d*. Whence $20x-320+8x=184d$.
7 That is, $28x-320=184$,
8 That is, $28x=184+320=504$.
9 Therefore $x=\frac{504}{28}=18$ Days *Anſ.*
10 And by 2*d* Step, $40-x=22$ idle.

Literal

Literal Solution.

Let $b=40$, $c=20$, $d=8$, and $f=184$ Pence, *viz.* 15*s.* 4*d.*

1 | x Days wrought,
2 | $b-x$
3 | cx
4 | $db-dx$. Whence
5 | $cx-db+dx=f$
6 | $cx+dx=f+db$
7 | $x=\frac{f+db}{c+d}=18$, as before, the days he worked
8 | $b-x=22$ idle.

PROOF.

	£.	s.	d.
18 Days, at 20*d.* a Day,	1	10	0
22 idle, at 8*d.* a Day,	0	14	8
Due at laſt	0	15	4

* PROBLEM VII.—*Cocker's 6th* Queſtion.

Two Perſons, A *and* B, *thus diſcourſed of their Money: Says* A *to* B, *give me* 3 *of your Crowns, and I ſhall have as many as you; and ſays* B *to* A, *give me* 3 *of yours, and I ſhall have* 5 *Times as many as you. I demand the Number each had?*

Numerical Solution.

1 | For the N°. *A* had at firſt put x,
2 | Then by *B*'s giving him 3, he will have $x+3$;
3 | And as this makes *B*'s equal, *B* will alſo have $x+3$;
4 | And therefore *B* had at firſt $x+6$;
5 | And if *A* had given him 3, he would then have $x+9$,
6 | And by the ſame Reaſon *A* would have had but $x-3$:

7 Now *B* (Q.) ſhould have 5 Times this Number, *viz.* $5x - 15$;
8 Whence this *Equation*, $5x - 15 = x + 9$.
9 Then φ x and -15, it will be $5x - x = 9 + 15$,
10 That is $4x = 24$.
11 Therefore $x = \frac{24}{4} = 6$, *A*'s Number of Crowns;
12 And by the 4*th* Step $x + 6 = 12$, *B*'s.

Literal Solution.

Let $e = 3$ Crowns; then by comparing the Steps,

1 x for *A*,
2 $x + e$,
3 $x + e$,
4 $x + 2e$,
5 $x + 3e$,
6 $x - e$,
7 $5x - 5e$
8 $5x - 5e = x + 3e$,
9 $5x - x = 3e + 5e$,
10 $4x = 8e$
11 $x = \frac{8e}{4} = 2e = 6$, *A*'s.
12 $x + 2e, = 12$, *B*'s.

PROOF.

A $6+3 = 9$,
B $12-3 = 9$, but
A $6-3 = 3$, and
B $12+3 = 15 = 3 \times 5$

Nov. I am perfectly ſatisfied, it is done ſo plain, and the Steps are built upon Reaſon itſelf.

Phi. It is upon Reaſon itſelf that *Algebra* depends; and a Queſtion laid down in a good and clear Light, is the Learner's chief Guide. But now for ſome that require *Fractions*, for I ſuppoſe you underſtand them!

Nov. Yes, I think I do pretty well.

Phi. Here follows then

* PROBLEM VIII.—*Cocker's* 5th.

There is a Fiſh whoſe Head is 9 *Inches long, and his Tail is as long as his Head, and Half as long as his Body, and his Body is as long as his Tail and his Head; I demand the whole Length of the Fiſh.*

1 For the Length of his Body put x
2 Then will his Tail be $\frac{x}{2} + 9$,
3 And his Body ſhould be as long as his Tail and Head, *viz.* $\frac{x}{2} + 9 + 9$,
4 Whence (Q.) this Equation $x = \frac{x}{2} + 18$,
5 This × Denominator 2, is $2x = x + 36$.
6 Then φ x, $2x - x = 36$
7 That is, $x = 36$ his Body,
8 And by 2*d* Step, $\frac{x}{2} + 9 = 27$ his Tail;
9 To which add 9 his Head.
10 Their Sum is 72 his Length.

Literal Solution.

Let $b = 9$,
1 x,
2 $\frac{x}{2} + b$
3 $\frac{x}{2} + 2b$
4 $x = \frac{x}{2} + 2b$
5 $2x = x + 4b$.
6 $2x - x = 4b$.
7 $x = 4b = 36$, Body.
8 $\frac{x}{2} + 6 = 27$, Tail.

* PROBLEM IX. His 12*th* Queſtion.

One aſked a Shepherd the Price of his 100 *Sheep. No, replied he, if I had as many more, and Half as many more, and ſeven Sheep and an Half, I ſhould then have* 100. *I demand the Number he had.*

Numerical Solution.

Put
1 x for the Number, then
2 x as many more, and
3 $\frac{x}{2}$ is Half as many more
4 $7\frac{1}{2}$. Theſe four Steps added
5 $2x + \frac{x}{2} + 7\frac{1}{2}$. Whence (Q.)
6 $2x + \frac{x}{2} + 7\frac{1}{2} = 100$. This reduced
7 $4x + x + 15 = 200$. Then φ 15,
8 $5x = 200 - 15 = 185$. Therefore
9 $x = \frac{185}{5} = 37$ *Anſwer.*

Literal Solution.

Let $b = 7\frac{1}{2}$, $c = 100$.

1 x,
2 x,
3 $\frac{x}{2}$,
4 b,
5 $2x + \frac{x}{2} + b$ Sum (Q.)
6 $2x + \frac{x}{2} + b = c$,
7 $4x + x + 2b = 2c$,
8 $5x = 2c - 2b$,
9 $x = \frac{2c - 2b}{5} = 37$. *Anſ.*

* PROBLEM X. His 9*th* Queſtion.

A Gentleman bought a Cloak of a Saleſman, which coſt him 3*l.* 10*s. and after he had bought it he deſired the Saleſman to tell him ingenuouſly what he gained by it, who anſwered I gain juſt* ¼ *Part of what it coſt me. It is demanded what the Cloak coſt the Saleſman.*

Numerical Solution.

1 For what the Cloak coſt the Saleſman put x;
2 Then his Gain or Profit will be $\frac{x}{4}$;
3 Theſe added are equal to what he ſold it for, $x+\frac{x}{4}$.
4 Therefore (Q.) $x+\frac{x}{4}=70s.$
5 This reduced, $4x+x=280$:
6 That is, $5x=280$.
7 Therefore $x=\frac{280}{5}=56s.$ *Anſ.*

Literal Solution.

Let $e=70$.

1 x,
2 $\frac{x}{4}$.
3 $x+\frac{x}{4}$
4 $x+\frac{x}{4}=e$,
5 $4x+x=4e$,
6 $5x=4e$,
7 $x=\frac{4e}{5}=56$ as before

PROOF.

Coſt him 56 *s.*
Gained $\frac{1}{4}$ = 14 *s.*

Sum = 70 *s.* it ſold for

* PROBLEM XI. His 11*th* Queſtion.

A Perſon in the Afternoon being aſked what o'Clock it was, anſwered, that $\frac{3}{5}$ of the Time paſt from Noon was equal to $\frac{5}{8}$ of the Time to Midnight. Now, allowing 24 Hours to the Day, and beginning to reckon from Noon, I demand what Hour it was when the Queſtion was aſked.

Numerical Solution.

1 For the Hours ſought from Noon put x
2 Then will the Time to Midnight be $12 - x$,
3 Now $\frac{3}{5}$ of x is $\frac{3x}{5}$,
And $\frac{5}{8}$ of $12 - x$ is $\frac{60 - 5x}{8}$
5 Theſe (Q.) are equal. Whence $\frac{3x}{5} = \frac{60 - 5x}{8}$
6 This reduced, *viz.* firſt × Denominator 5, is $3x = \frac{300 - 25x}{8}$
7 Then this × the Denominator 8 will be $24x = 300 - 25x$
8 Then $\varphi - 25x$ it is $24x + 25x = 300$,
9 That is, $49x = 300$.
10 Therefore $x = \frac{300}{49} = 6\frac{6}{49}$
11 And by 2*d* Step $12 - x = 5\frac{43}{49}$
So that it was $\frac{6}{49}$ paſt 6, that is 7′ 20″ $\frac{40}{49}$ paſt 6.

PROOF.

H.

$$6\tfrac{6}{49} = 6 \quad 7 \quad 20\tfrac{40}{49}$$
$$5\tfrac{43}{49} = 5 \quad 52 \quad 39\tfrac{9}{49}$$

Sum = 12 Hours.

Literal Solution.

$$\text{Let } b = \frac{3}{5},\ c = \frac{5}{8}, \text{ and } d = 12$$

1 | x as above,
2 | $d - x$,
3 | bx,
4 | $cd - cx$
5 | $bx = cd - cx$,
6 | $bx + cx = cd$,
7 | $x = \dfrac{cd}{b+c}$, viz. $x = \dfrac{5}{8} \times 12 \div \overline{\dfrac{3}{5} + \dfrac{5}{8}} = 6\tfrac{48}{392}$ or $6\tfrac{6}{49}$ as before.

Nov. This is plain, upon my Word, and I perceive $6\frac{48}{392}$ is the ſame Anſwer as Mr. *Cocker*'s; but ſtill I never could know, nor do I yet rightly apprehend from whence this $\frac{48}{392}$ proceeds, and therefore I never could make the Anſwer chime in with his.

Phi. I muſt own that a Man had need to underſtand *Fractions* quite perfectly to find ſuch Things out to his own Satisfaction; for it is not every *Learner* can do it. Obſerve then, the Anſwer is $x = \frac{5}{8} \times 12 \div \frac{3}{5} + \frac{5}{8}$. Now $\frac{5}{8} \times \frac{12}{1} = \frac{60}{8}$. And $\frac{3}{5} + \frac{5}{8}$ being reduced to a common Denominator and added, their Sum will be $\frac{24}{40}$ and $\frac{25}{40}$ (that is $\frac{49}{40}$) by which divide $\frac{60}{8}$ it is $\frac{2400}{392} = 6\frac{48}{392}$ as before; and therefore the Hour to Midnight $5\frac{344}{392} = 5\frac{43}{49}$.

Tyr. There, *Novitius*, is that plainly demonſtrated or not?

Nov. Quite plain indeed! And now, *Philomathes*, I will aſk you to work the 6*th* Queſtion and no more; for the 5*th* Step I never could apprehend, it has puzzled me many a Time.

Phi. I muſt confeſs I was a long Time in finding it out myſelf, and though the ſame Queſtion be in Mr. *Ward*, it did not anſwer my Expectation; for I found it out by Mr. *Cocker*'s Method at laſt.

Nov. But then he abbreviates his Numbers and *Fractions*, and gives no Reaſon, and this puts the Learner to a Stop.*

Phi. It is true he does ſo. Well, *Novitius*, all I can ſay, is, I will work the ſame Queſtion *numerically*, and leave you to judge which of the three Ways are the eaſieſt to be underſtood, ſuppoſing you had them now all before you.

* PROBLEM XII. *Cocker*'s *6th*, and *Ward*'s *31ſt* Queſtion.

A Father lying at the Point of Death, left his three Sons, A, B, *and* C, *his Eſtate as follows. To* A *he gave* $\frac{1}{2}$ *wanting* 44*l.* *to* B *he gave* $\frac{1}{3}$ *and* 14*l.* *over; and to* C *he gave the Remainder, which was* 82*l.* *leſs than the Share of* B. *I demand the Father's Eſtate in ready Money.*

Numerical Solution.

1 | For the Eſtate put x,

2 | Then will *A*'s Legacy be $\frac{x}{2} - 44$,

3 | And *B*'s will be $\frac{x}{3} + 14$,

4 | And *C*'s being 82 leſs, is $\frac{x}{3} + 14 - 82$.

5 | Theſe Fractions being firſt reduced to a common Denominator, and added, the Sum of the Whole is $\frac{21x}{18}$ or $\frac{7x}{6} + 28 - 126$. This (Q.) equal to the Eſtate or x.

6 | Whence this Equation $\frac{7x}{6} + 28 - 126 = x$.

---

* See the Preface.

7 Now $28 - 126$ being the ſame as -98, it will be $\frac{7x}{6} - 98 = x$.

8 This reduced $7x - 588 = 6x$.

9 Then φ $6x$, and 588, it will be $7x - 6x = 588$,

10 That is, $x = 588$ Eſtate.

A's Share $\frac{1}{2} - 44 = 250 = \frac{x}{2} - 44$ by 2*d* Step.

B's $\frac{1}{3} + 14 = 210 = \frac{x}{3} + 14$ by 3*d* Step.

C's 82*l.* leſs than $B = 128 = \frac{x}{3} + 14 - 82$ by 4*th* Step.

588 The Eſtate

Nov. I heartily thank you, kind *Philomathes*; then I perceive that Mr. *Cocker*'s 5*th* Step before *Abbreviation* was $\frac{21a}{18} + 2c - b - d$.

Phi. You are right, and one would think a Learner might eaſily perceive it if he would be diligent.

Nov. You know, Sir, a ſmall Matter turns the Learner quite out of his direct Path. I have now with me *James de Billy*'s *Algebra*, but it is wrought in ſo odd a Manner, that I cannot make out any Thing by it. I could wiſh you would work ſome of his Queſtions *numerically*.

Phi. Which would you have me begin with?

Nov. His 2*d* Queſtion, if you pleaſe.

* PROBLEM XIII.—*J. de Billy*'s 2*d* Queſtion.

A Hare is 100 Yards diſtant from a Dog, and both ſtarting together, the Dog ran 2½ Times faſter than the Hare: It is demanded how far the Hare will have run before the Dog overtakes her?

Numerica

Numerical Solution.

1 For the Yards the Hare ran put x,

2 Then will the Dog, when he overtakes her, have run $100 + x$.

3 Now becauſe he run $2\frac{1}{2}$ Times faſter than the Hare, take any two Numbers bearing the like Proportion as 5 and 2, then

4 By the Rule of 3, as $5 : 2 :: 100 + x : x$, †

5 Multiplying *Means* and *Extremes*, $5x = 200 + 2x$

6 Then φ $2x$, —————— $5x - 2x = 200$,

7 That is, —————— $3x = 200$.

8 Therefore $x = \frac{200}{3} = 66\frac{2}{3}$.

9 And by 2*d* Step, $100 + x = 166\frac{2}{3}$.

So that the Hare run $66\frac{2}{3}$ Yards, and the Dog $166\frac{2}{3}$.

† *Note*, From this Step we have,

$\overline{5 - 2} : 2 :: \overline{100 + x - x} : x$; *i. e.*

$3 : 2 :: 100 : \left(\frac{200}{3} =\right) x$. See Euclid, Book V. Def. 16, and Prop. 17. EDIT.

* PROBLEM XIV.—His 11*th* Queſtion.

A certain Man agreed with his Servant for 12 Months Service to give him 10 Crowns and a new Coat; but diſagreeing, he at the End of 7 Months gives him the Coat and two Crowns: I demand the Value of the Coat?

Numerical Solution.

1 For the Value of the Coat put x,

2 Then will this and 10 Crowns be his Year's Wages, $x + 10$.

3 Now to find one Month's Wages ſay,

As $12 : x + 10 :: 1 : \frac{x + 10}{12}$.

4 And becauſe he had 2 Crowns and the Coat for 7 Months, ſay,

As $7 : x + 2 :: 1 : \frac{x + 2}{7}$.

5 Theſe, being both 1 Month's Wages, are equal.

Whence $\frac{x + 2}{7} = \frac{x + 10}{12}$.

6 This × the Denominator 7, is $x + 2 = \frac{7x + 70}{12}$

7 This × 12, the other Denominator, $12x + 24 = 7x + 70$; then φ $7x$ and 24,

8 $12x - 7x = 70 - 24$, that is,

9 $5x = 46$,

10 Therefore $x = \frac{46}{5} = 9\frac{1}{5}$ Crowns.

So that the Value of the Coat was $9\frac{1}{5}$ Crowns, or 46 Shillings, to which add two Crowns, is 56 Shillings that he had for his 7 Months Service; and for the Year's Service, had he ſtaid, $x + 10$ Crowns by 2*d* Step, *viz.* 96 Shillings.

Note, This Queſtion may be very eaſily anſwered by common Arithmetic. For, as the Wages for 12 Months are 10 Crowns and a new Coat, and for 7 Months are 2 Crowns and the Coat, it is plain that 8 Crowns are the Wages for 5 Months. Then $5 : 8 :: 7 : \frac{56}{5} = 11\frac{1}{5}$ Crowns, the Wages for 7 Months; and if from this we take 2 Crowns, there will remain $9\frac{1}{5}$ Crowns, or 46 Shillings, the Value of the Coat.

EDIT.

* PROBLEM XV.—His 16*th* Queſtion.

A Man having a certain Number of Crowns about him, deſired a Stander-by to gueſs at them, who ſaid, you have 600 *perhaps. No, ſays he; but if to what I have were added* $\frac{1}{2}$, $\frac{1}{3}$, *and* $\frac{1}{4}$, *and from what I have were ſubtracted* $\frac{1}{12}$, *I ſhould then have* 600: *I demand the Number he had about him?*

Note, As in *Problem* 2, ſo here alſo make the Subtraction firſt, and work with the Difference only. Thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, reduced to a common Denominator, will be $\frac{26}{24}$, or $1\frac{2}{24} = 1\frac{1}{12}$; from which take $\frac{1}{12}$, there remains 1 whole Integer. So that ſuppoſing I were to put x for the Number, then $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ added, and $\frac{1}{12}$ taken from it, x will be the Difference; therefore $2x = 600$, and $x = 300$ the Number. But ſee the Whole as follows.

1 | For the Number ſought put x,

2 | Then $\frac{x}{2}, \frac{x}{3}, \frac{x}{4}, = \frac{26x}{24},$ that is, $= x + \frac{x}{12}$ as before.

3 | Add theſe, it is $2x + \frac{x}{12}$,

4 | From which take $\frac{1}{12}$, *viz.* $\frac{x}{12}$,

5 | There remains only $2x$. Whence (Q.) $2x = 600$.

6 | And therefore $x = 300$ *Anſ.*

Nov. I heartily thank you; this appears plain enough to me. I will trouble you to work one more, if you pleaſe, and that is his 7*th* Queſtion, for this is quite dark to me.

* PROBLEM XVI.—His 7*th* Queſtion.

A certain Perſon bought a Number of Ells of Velvet, which he ſold again; he bought 5 *Ells for* 7 *Crowns, and ſold ſeven Ells for* 11 *Crowns, and gained* 100 *Crowns in ſo doing: I demand how many Ells there were in all?*

Numerical Solution.

1 | For the Number put x,

2 | Then, if 5 Ells be 7 Crowns, what will x be? $\frac{7x}{5}$,

3 | Then if 7 Ells be ſold for 11 Crowns, what will x fetch? — — — $\frac{11x}{7}$,

4 The Difference of theſe ſhews his Gain $\frac{6x}{35}$ *
5 This (Q.) $= 100$ Crowns. Whence $\frac{6x}{35} = 100$
6 This reduced $6x = 3500$.
7 Therefore $x = \frac{3500}{6} = 583\frac{1}{3}$.
So that the Number was $583\frac{1}{3}$ Ells bought and ſold.

PROOF.

If 5 Ells be 7 Crowns, what is $583\frac{1}{3}$? *Anſ.* $816\frac{1}{3}$ Cr.
If 7 Ells be 11 Crowns, what is $583\frac{1}{3}$? *Anſ.* $916\frac{1}{3}$ Cr.

He gained 100 Cr.

* *Note* 1. The Fraction $\frac{7x}{5}$ and $\frac{11x}{7}$ are firſt reduced to a common Denominator, and then the Difference you will find is $\frac{6x}{35}$

Note 2. This Queſtion may be eaſily anſwered by Fractions, without Algebra. EDIT.

PROBLEM XVII.

There are two Numbers whoſe Sum is 240, *and the Greater has the ſame Proportion to the Leſs as* 7 *to* 3: *I demand the Numbers?*

Numerical Solution.

1 For the Greater put x,
2 Then will the Leſs be $240 - x$.
3 Theſe (Q.) have this Proportion,
as $7 : 3 :: x : 240 - x$.
4 Then multiplying *Means* and *Extremes*,
$3x = 1680 - 7x$.
5 Then $\varphi - 7x$, it is $3x + 7x = 1680$.
6 That is, $10x = 1680$.
7 Therefore $x = \frac{1680}{10} = 168$ G. Number,
8 And by 2*d* Step $240 - x = 72$ L. Number.

Note, This Queſtion may be anſwered by common Arithmetic, as follows:

As 7 : 3 : : Gr. No. : the leſs; and therefore,
As 10 : 3 : : Sum of the Numbers : the leſs; i. e.
As 10 : 3 : : 240 : $\frac{720}{10} = 72$. EDIT.

* PROBLEM XVIII.

A certain Toper went to an Alehouſe, and borrowed as much Money as he had about him, out of which he ſpent a Shilling; then he went to a 2d Alehouſe, and borrowed as much as he had then about him, and ſpent a Shilling; and in like Manner he went to a 3d and 4th Alehouſe, borrowing as much as he had left at the former, and ſpent a Shilling; but after he had ſpent a Shilling at the 4th Alehouſe he had nothing left: It is demanded what he had firſt about him?

Numerical Solution.

1 For what he firſt had put x,
2 Then by borrowing as much he had $2x$,
3 And when he had ſpent 12 Pence, had $2x - 12$.
4 Then by borrowing the ſame, had $4x - 24$,
5 And by ſpending 12 Pence, had $4x - 36$;
6 Then, by borrowing the ſame, had $8x - 72$,
7 And by ſpending 12 Pence, had $8x - 84$.
8 Then by borrowing the ſame, had $16x - 168$,
9 Then by ſpending 12 Pence, had left $16x - 180$,
10 That is, had nothing left. Whence (Q.) $16x - 180 = 0$,
11 That is, $16x = 180$.
12 Therefore $x = \frac{180}{16} = 11d.\ \frac{1}{4}$

So that he had at firſt 11*d*. $\frac{1}{4}$.

Note, This Queſtion may be anſwered without Algebra, as follows: Since he had no Money left after he had paid 1*s*. at the 4*th* Alehouſe, where he borrowed $\frac{1}{2}$, *i. e.* 6*d*. it is evident that he brought 6*d*. into that Houſe, and conſequently had 6*d*. left after he had paid 1*s*. at the 3*d* Alehouſe. And by reaſoning in this

Manner we may eaſily find what he brought into the *2d* and *1ſt* Houſes.

Left at 3*d* Houſe	6*d*.
Had before the Reckoning was paid	18*d*.
The $\frac{1}{2}$ of which was borrowed	9*d*.
Therefore he brought from 2*d* Houſe	9*d*.
And had before Payment	21*d*.
The $\frac{1}{2}$ of which was brought from 1*ſt* Houſe	10*d*. $\frac{1}{2}$
Where he had before Payment	22*d*. $\frac{1}{2}$

But as Half of this was borrowed, it is evident that he carried into that Houſe 11*d*. $\frac{1}{4}$. EDIT.

* PROBLEM XIX.

One being aſked how old he was, anſwered thus:

If to the Number of my Age you add
The one Half of three Fourths, and 14 *more,*
The Number 58 *will then be had:*
What is my Age in Years above a Score?

Note, As the Queſtion ſays, if you add 14 to his Age, it will make 58; ſo conſequently without adding the 14 it will be 44; therefore working with 44, without adding 14, will be better than to work with 58 and 14 together, as you may obſerve in *Problem 3d.*

Numerical Solution.

1 For his Age put x,

2 Then by adding $\frac{1}{2}$ of $\frac{3}{4}$, that is, $\frac{3}{8}$, it is $x + \frac{3x}{8}$.

This (Q.) $= 44$. Whence $x + \frac{3x}{8} = 44$.

3 This reduced $8x + 3x = 352$.

4 That is, $11x = 352$.

5 Therefore $x = \frac{352}{11} = 32$.

6 So that his Age was 32, 12 above a Score.

PRO-

PROBLEM XX.

A Citizen riding his Rounds to receive Money due to him, came to a Place in which he had three Debtors, A, B, *and* C, *but when he came to examine, he found he had loſt his Pocket-Book, in which was each Man's ſeparate Bill. Then ſending for them all to an Inn, he tells them the Accident; but they pretended they did not know what was due to him, having loſt the Bills that came with the Parcels. The Gentleman thinking he had got ſome ſlippery Chaps to deal with, endeavoured all he could to ſave himſelf the Trouble of another Journey; and this he did from the following* Data: *He remembered very well that* A's *and* B's *Debt together made* 13*l.* 10*s. and* A's *and* C's *Debt together* 31*l.* 10*s. and* B's *and* C's *together made* 37*l.* 10*s. It is demanded what each Man's particular Debt was?*

1 For *A*'s Debt put x,

2 Then as *A*'s and *B*'s together is 270*s*. *B*'s is $270 - x$,

3 Then as *A*'s and *C*'s is 630*s*. *C*'s alone is $630 - x$.

4 Now (Q.) *B*'s and *C*'s ſhould be 750*s*. but is $900 - 2x$.

5 Whence this Equation $750 = 900 - 2x$.

6 By $\varphi\ 2x$, — — — $750 + 2x = 900$.

7 And $\varphi\ 750$, — — — $2x = 900 - 750$,

8 That is, $2x = 150$.

9 Therefore $x = \frac{150}{2} = 75s$. *A*'s

10 And by 2*d* Step, $270 - x = 195$, *B*'s.

11 And by 3*d* Step, $630 - x = 555$, *C*'s.

	£.	*s.*
So that *A*'s Debt was	3	15
B's — — —	9	15
C's — — —	27	15

PROBLEM XXI.

A Highwayman robbed a Gentleman of a certain Sum of Money, but being ſeen by three Men, A, B, *and* C, *they purſued and took him; but he promiſed to make them a handſome Preſent if they would let him go; to which they agreed. To* A *he gave* $\frac{1}{2}$, *and* A *returned him back 6l. And to* B $\frac{1}{3}$ *of what was left, who returned him 4l. And to* C *he gave* $\frac{1}{4}$ *of what he had then left, who returned him back 2l. And after he had rode off, and came to tell his Money, he found he had given* $\frac{2}{3}$ *of it away: I demand the Sum he took from the Gentleman?*

Numerical Solution.

1 For what he took in Pounds put x,

2 Then by giving $\frac{1}{2}$ he had the ſame left, $\frac{x}{2}$

3 And by *A*'s returning him 6, he had $\frac{x}{2}+6$

4 This he gave *B* $\frac{1}{3}$ of, *viz.* $\frac{x}{6}+2$,

5 And conſequently had himſelf $\frac{2}{3}$, *viz.* $\frac{2x}{6}+4$, or $\frac{x}{3}+4$,

6 And by *B's* returning 4, had $\frac{x}{3}+8$

7 This he gave *C* $\frac{1}{4}$ of, *viz.* $\frac{x}{12}+2$,

8 And conſequently had in Hand $\frac{3}{4}$, *viz.* $\frac{3x}{12}+6$,

9 And by *C*'s returning 2, had $\frac{3x}{12}+8$, or $\frac{x}{4}+8$,

10 This (Q.) = $\frac{1}{3}$ of what he ſtole. Whence $\frac{x}{3}=\frac{x}{4}+8$.

11 Then multiplying by the Denominator 3, $x = \frac{3x}{4} + 24$.

12 This × the Denominator 4, $4x = 3x + 96$.

13 Then ϕ $3x$ it will be $4x - 3x = 96$,

14 That is $x =$ £. 96 *Anſ.*

So that he ſtole £. 96; of which *A* had 42, *B* 14, and *C* £. 8, which together make £. $64 = \frac{2}{3}$ of 96.

PROBLEM XXII.

One being aſked how many Children he had living, anſwered 3 Times as many as he had buried; and being aſked how many that was, ſaid, that if the Number he had loſt was multiplied by the $\frac{1}{9}$ Part of what remained, it would be equal to the Number he had at firſt: I demand how many he had loſt, and how many he had left?

Numerical Solution.

1 For what he had loſt put x,

2 Then will the living be $3x$,

3 And conſequently he had at firſt $4x$.

4 Now $\frac{1}{9}$ of what was left is $\frac{3x}{9}$, or $\frac{x}{3}$.

5 This multiplied by the loſt, *viz.* x, is $\frac{xx}{3}$.

6 This (Q.) equals his Number at firſt. Whence $\frac{xx}{3} = 4x$.

7 This reduced, $xx = 12x$.

8 Then dividing by x, ——— $x = 12$, loſt.

9 And by 2*d* Step, $3x = 36$, living.

10 Their Sum by 3*d* Step is $4x = 48$, at firſt.

PROBLEM XXIII.

One being aſked how many Teeth he had, to avoid a direct Anſwer ſaid, that he had loſt the $\frac{1}{8}$ *Part of what he then had, and being aſked how many that was, ſaid, that if what he had loſt were multiplied into the* $\frac{1}{4}$ *of what he had left, and the Square of what he had loſt was added to that Product, it would be equal to the Number he had at firſt. Or otherwiſe, if what he had loſt were multiplied by* $\frac{1}{4}$ *of what he had left, it would make juſt* $\frac{2}{3}$ *of the Number he had at firſt: I demand how many he had loſt, and how many he had left?*

Numerical Solution.

1 For the Number loſt put x,
2 Then as thoſe left are 8 Times as many, $8x$,
3 Theſe two added make the Number at firſt $9x$.
4 Now $\frac{1}{4}$ of the Number left is $2x$,
5 This multiplied by thoſe loſt, make $2xx$,
6 To which add the Square of thoſe loſt, it is $2xx + xx$.
7 Theſe (Q.) are equal to the Number at firſt.
8 Then dividing each Side by x, $2x + x = 9$.
9 That is $3x = 9$.
10 Therefore $x = 3$, loſt.
11 And by 2*d* Step, $8x = 24$, left.
12 And by 3*d* Step, $9x = 27$, at firſt,
Which you may prove according to the Tenor of the Queſtion.

PROBLEM XXIV.

An Uſurer put out a certain Sum of Money at 5*l.* per Cent. per Annum, *which in* 16 *Years wanted exact* 11 *Guineas of the Principal itſelf: I demand what the Principal was.*

Numerical Solution.

1 For the Principal put x,
2 Then ſay, If 100 give 5, what will x give? *Anſ.* $\frac{5x}{100}$

3 This being 1 Year's Intereſt, 16 Years is $\frac{80x}{100}$, or $\frac{8x}{10}$, or $\frac{4x}{5}$.

4 This ſhould (Q.) be equal to the Principal leſs 11 Guineas, or 231 Shillings. Whence,

$$x - 231 = \frac{4x}{5}.$$

5 This being reduced, $5x - 1155 = 4x$,

6 Then φ 4x, — — — $5x - 4x = 1155$,

7 That is $x = 1155s. = £.\ 57,\ 15s.$

So that the Principal was 57*l.* 15*s.* which in 16 Years, at 5 *per Cent.* amounts to 46*l.* 4*s.* which wants 11 Guineas of the Principal.

PROBLEM XXV.

An Uſurer put out 135*l. in* 2 *Parcels, one at the Rate of* 5 per Cent. per Annum, *and the other at* 6 per Cent. *which amounted in* 15 *Years Time to the Principal itſelf wanting* 30*l. I demand the Parcel he put out at* 5, *and the Parcel he put out at* 6 per Cent.

Numerical Solution.

1 For the Parcel at 5 *per Cent.* put x,

2 Then will that at 6 *per Cent.* be $135 - x$,

3 Then by the Rule of 3, one Year's Intereſt of x, at 5 *per Cent.* is $\frac{5x}{100}$,

4 And by the ſame Rule, one Year's Intereſt of $135 - x$ at 6 *per Cent.* is $\frac{810 - 6x}{100}$

5 The Sum of the Intereſt of both for 1 Year is $\frac{5x}{100} + \frac{810 - 6x}{100}$

6 Then becauſe there is $5x$ and $-6x$, it will be $\frac{810 - x}{100}$.

7 This being 1 Year's Intereſt of both, 15 Years will be $\frac{12150 - 15x}{100}$.

8 This (Q.) is equal to 135 − 30*l.* *viz.* 105*l.*
Whence $105 = \frac{12150 - 15x}{100}$

9 This × the Denominator 100, is
$10500 = 12150 - 15x$.

10 Then $\varphi - 15x$ and 10500,
$15x = 12150 - 10500$,

11 That is, $15x = 1650$.

12 Therefore, $x = \frac{1650}{15} =$ £. 110

13 And by 2*d* Step, $135 - x =$ £. 25
So that he put out £. 110 at 5, and £. 25 at 6 *per Cent.* the Intereſt of which in 15 Years amounts to £. $105 = 135 - 30$.

PROBLEM XXVI.

It is required to pay £. 100 in 100 Pieces, viz. *ſome to be 15 Shillings, and others 22 Shillings and 6 Pence each: I demand how many there muſt be of each Sort?*

Numerical Solution.

1 Put for the 15*s.* Pieces x,
2 Then will thoſe at 22*s.* 6*d.* be $100 - x$,
3 Now x Pieces, at 15*s.* or 180*d.* each is $180x$.
4 And $100 - x$ Pieces at 22*s.* 6*d.* or 270*d* each, is $27000 - 270x$.
5 The Sum of theſe two is $27000 - 90x$.
6 This (Q.) = £. 100, or 24000 *d.* Whence
$24000 = 27000 - 90x$
7 Then $\varphi - 90x$, $90x + 24000\,d. = 27000$.
8 Then φ 24000, $90x = 27000 - 24000$,
9 That is, $90x = 3000$,
10 That is, $9x = 300$.
11 Therefore, $x = \frac{300}{9} = 33\frac{1}{3}$,
12 And by 2*d* Step, $100 - x = 66\frac{2}{3}$.
And ſince x is not a whole Number, it is evident that £. 100 cannot be paid in 100 Pieces of the Value ſtated in the Queſtion.

PROBLEM XXVII.

A Vintner has two Veſſels full of Wine, equal alike in Quantity, but of different Quality; the worſt Sort is worth 240 Crowns, and the beſt 300: Now he has another Caſk or Veſſel of the ſame Size, which he intends to fill out of theſe two, that when full may be worth 260 Crowns: How much of each muſt he take?

Numerical Solution.

1 For what he muſt take of the worſt Sort put x,

2 Then as both make but one Veſſel, the beſt will be $1 - x$.

3 Now if 1 full Veſſel be worth 240 Crowns, x of a Veſſel is worth $\frac{240x}{1}$

4 And by the ſame Rule, $1 - x$ of a Veſſel is worth $\frac{300 - 300x}{1}$

5 The Sum of the Numerators of the 3*d* and 4*th* Step is $300 - 60x$

6 This (Q.) equal to the mean Price, 260 Crowns. Whence this Equation, $260 = 300 - 60x$.

7 Then φ 260 and $- 60x$, $60x = 300 - 260$.

8 That is, —— —— —— $60x = 40$.

9 Then muſt $6x = 4$. Therefore,

10 $x = \frac{4}{6} = \frac{2}{3}$. And by 2*d* Step,

11 $1 - x = \frac{1}{3}$.

So that he muſt take $\frac{2}{3}$ of the worſt, and $\frac{1}{3}$ of the beſt Sort.

PROOF.

If 1 Veſſel be worth 240 Crowns, $\frac{2}{3}$ will be worth — — } 160

And if 1 Veſſel be worth 300, then $\frac{1}{3}$ will be worth — — } 100

Sum 260 = M. Price.

PRO-

PROBLEM XXVIII.

Two Men, A *and* B, *ſet out from a certain Place, the one goes* 21 *Miles in* 15 *Hours, and* 8 *Hours after he ſet out,* B *begins to travel, and goes at the Rate of* 15 *Miles in* 9 *Hours; I demand how long it will be before* B *overtakes* A, *and how far they will both have travelled?*

Literal Solution.

1 For the Hours *A* travelled put x,

2 Then will *B* travel $x - 8$.

3 Then, If 15 Hours give 21 Miles, what will x give? *Anſ.* $\frac{21x}{15}$

4 And, if 9 Hours give 15 Miles, $x - 8$ will give $\frac{15x - 120}{9}$

5 Now ſeeing that after *B* overtakes *A*, the Diſtances they travelled were both alike, therefore $\frac{15x - 120}{9} = \frac{21x}{15}$ *

6 This reduced firſt by 9, is $15x - 120 = \frac{189x}{15}$.

7 This × 15, is $225x - 1800 = 189x$.

8 Then φ $189x$, $225x - 189x = 1800$,

9 That is, $36x = 1800$,

10 That is $x = \frac{1800}{36} = 50$ Hours, *A*.

11 And by the 2*d* Step, $x - 8 = 42$ Hours, *B*,

12 Now *A* goes by 3*d* Step $\frac{21x}{15}$, that is,

70 Miles $= \frac{15x - 120}{9} = 70$, *B*.

---

* *Note.* $\frac{15x - 120}{9} = \frac{5x - 40}{3}$; and $\frac{21x}{15} = \frac{7x}{5}$; and the 5*th* Step becomes $\frac{5x - 40}{3} = \frac{7x}{5}$, which reduced is $25x - 200 = 21x$; and then φ $21x$ and -200, we have, $25x - 21x = 200$, or $4x = 200$, and $x = \frac{200}{4} = 50$. EDIT.

PROBLEM XXIX.*

What Number is that *that $\frac{3}{4}$ of it more* 12 *is equal to $\frac{2}{3}$ of it more* 14*?*

1 For the Number ſought put x

2 Then $\frac{3}{4}$ of it + 12, is $\frac{3x}{4} + 12$.

3 And $\frac{2}{3}$ of it + 14, is $\frac{2x}{3} + 14$.

4 Theſe (Q) are equal.

Whence $\frac{3x}{4} + 12 = \frac{2x}{3} + 14$.

5 This reduced firſt, is $3x + 48 = \frac{8x}{3} + 56$

7 That is, being again reduced,
$9x + 144 = 8x + 168$.

8 Then φ $8x$ and + 144, $9x - 8x = 168 - 144$,
That is, $x = 169 - 144 = 24$ *Anſ.*
And this is proved at large in *Dial.* 8. *Sect.* 4. *Ex.* 5.

Note, From the 4th Step, $\varphi + 12$ we get

$$\frac{3x}{4} = \frac{2x}{3} + 2\text{; and then, by Reduction,}$$

$$3x = \frac{8x}{3} + 8$$

And $9x = 8x + 24$

And laſtly, $x = 24$. EDIT.

PROBLEM XXX.

Four Highwaymen, A, B, C, *and* D, *robbed a Gentleman upon the Road of* 475 £. *and going to an Inn to part the Money, which they had laid upon the Table, Words aroſe, and every one ſnatched up what he could; after which, upon telling each one his Money, it was found, that if to what* A *ſnatched up were added* 4 £. *and from* B's *were taken* 4, *and* C's *multiplied by* 4, *and* D's *divided by* 4, *it would produce one and the ſame Number of Pounds: It is demanded what each ſnatched up?*

---

* See the Note in the Proof of *Example* 5, in *Multiplication* of *Equations*.

☞ Firſt, Suppoſe *A* ſnatched x Pounds, then having 4 added, it would be $x + 4$. Now ſince by ſubſtracting 4 from *B*'s, his would be equal to *A*'s, he muſt have ſnatched $x + 8$; and ſince by multiplying *C*'s by 4, he would have the ſame, he muſt of courſe have ſnatched $\frac{x}{4} + 1$, which × 4, produces $x + 4$; and as *D*'s is to be the ſame if divided by 4, he muſt have ſnatched $4x + 16$.

Numerical Solution.

1 For what *A* ſnatched put x,
2 Then will *B*'s be as above — — $x + 8$,
3 *C*'s — — — — — — — $\frac{x}{4} + 1$,
4 *D*'s — — — — — — — $4x + 16$,
5 The Sum is — — — — $6x + \frac{x}{4} + 25$.
6 This (Q.) is equal to the Robbery.
Whence, $6x + \frac{x}{4} + 25 = 475$; or $6x + \frac{x}{4} = 450$.
7 This reduced, $24x + x = 1800$.
8 That is $25x = 1800$.
9 Therefore, $x = \frac{1800}{25} = 72$, *A*,
10 And by 2*d* Step, $x + 8 = 80$, *B*,
11 And by the 3*d*, $\frac{x}{4} + 1 = 19$, *C*,
12 And by the 4*th*, $4x + 16 = 304$, *D*,
Sum 475.

Tyr. This ſeems to be a very hard Queſtion at firſt reading.

Phi. It may be ſo; and yet in other Words it may be eaſy; for it is no other than this: What 4 Numbers muſt 475 be divided into, ſo that the *1ſt* having 4 added to it, the *2d* 4 taken from it, the *3d* multiplied by 4, and the *4th* divided by 4, may be all equal? Which Numbers are as before, *viz.* 72, 80, 19, and 304; which you may prove.

PROBLEM XXXI.

Two Graziers, A *and* B, *coming from a Fair, were met by two Highwaymen, who robbed* A *of* 25*l.* 10*s. and* B *of* 7*l.* 10*s. but upon their complaining that they had a great many Miles to ride, and Nothing to bear their Charges, he that robbed* A *returned him a certain Sum, and ſo did the other to* B. *Now after* A *come to tell* B *what he had left, and* B *come to tell* A, *it was diſcovered that they robbed* A *of three Times as much as* B, *and left* B $\frac{1}{5}$ *of what they left* A: *I demand what each was really robbed of?*

Numerical Solution.

1 For what they took from *B* put x,
2 Then will what they took from *A* be $3x$
3 Now *B* had 7*l.* 10*s.* or 150*s.* at firſt,
But now, — — — $150 - x$ left,
4 Alſo *A* had 25*l.* 10*s.* 510*s.* at firſt,
But now, — — — $510 - 3x$ left.
5 This (Q.) ſhould be 5 Times what *B* had left.
Whence, — $510 - 3x = 750 - 5x$
6 Then φ − $5x$, — $510 + 5x - 3x = 750$.
7 Then φ 510, — $5x - 3x = 750 - 510$.
8 That is, —— —— $2x = 240$.
9 Therefore, — $x = \frac{240}{2} = 120s.$ *B.*
10 And by the *2d* Step, — $3x = 360$, *A.*
So that they took 18*l.* from *A*, and 6 from *B*.

PROBLEM XXXII.

A General of an Army had a certain Number of Men, which he intended to place in a ſquare Battalia, but diſpoſing of them in Rank and File, found he had 90 *Men to ſpare; now thinking to get theſe in alſo, he enlarged his Square to one Man more in Rank and File, but then found he wanted* 39 *Men to complete the Square: What Number of Men had he, and how many ſtood in Rank and File?*

Numerical Solution.

1 For the N°. that ſtood in Rank and File put x.
2 Then will the Square of theſe be xx,
3 But having 90 to ſpare, he had $xx + 90$.
4 Now encreaſing Rank and File by 1, the Side is — — — $x + 1$,
5 The Square of which is — $xx + 2x + 1$,
6 From this take $xx + 90$, there remains $2x - 89$.
7 This (Q.) is equal to 39.
Whence, — — $2x - 89 = 39$,
8 Then φ — 89, — $2x = 39 + 89$,
9 That is — — $2x = 128$,
10 Therefore, — — $x = 64$ a Side.

Note. Mr. BENJ. DONN obſerves, in his *Eſſays* on *Vulgar* and *Decimal Arithmetic*, (2d Edit. pa. 221) *that the Difference of any two Square Numbers whoſe Roots differ by* 1, *is an odd Number, and equal to twice the leſſer Root more* 1, *or equal to the Sum of the Roots.* This is evident from what is done above. Conſequently, if n be put for any Number, and the Square of it be taken from the Square of $n + 1$, the Remainder will be $nn + 2n + 1 - nn$; i. e. $2n + 1$; and if this Remainder be put $= d$, we ſhall have $2n + 1 = d$, $n = \frac{d - 1}{2}$, and $n + 1 = \frac{d + 1}{2}$. *Example* 1. Let $d = 25$; then will $n = \frac{24}{2} = 12$. *Ex*. 2. Let $d = 129$, which is the Difference of the Squares in the above Queſtion; then $n = \frac{128}{2} = 64$, as before.

EDIT.

PROOF.

There were 64 Men in Rank, and 64 in File, and $64 \times 64 = 4096$, to which add 90, it is 4186. But had 65 Men been the Side of the Square, there would have been 4225 Men, which is 39 more than 4186, Q. E. D.

*PROBLEM XXXIII.

There is a Veſſel partly empty, in which are 20 Gallons of Wine, worth 8s. per *Gallon; now if it be filled up with Water, the Wine and the Water together will be worth* 6s. *a Gallon, and the Whole worth the ſame Money as when it was all Wine: I demand then what the Veſſel holds when full, or, which is the ſame, how many Gallons of Water will fill it up.*

Numerical Solution.

1 For what the Veſſel wants in Gallons put x.
2 Now 20 Gallons of Wine, at 8*s.* is 160*s.*
3 Now when the Veſſel is filled up, the Whole will fetch the ſame as the Whole of the Wine did, *viz.* 160*s.*
4 Then ſay, If $20 + x$ be 160*s.* what is 1 Gallon? — — *Anſ.* $\frac{160}{20 + x}$,
5 This (Q.) is equal to 6*s.* Whence, $6 = \frac{160}{20+x}$,
6 That is, — — $120 + 6x = 160$.
Then φ 120, — $6x = 160 - 120 = 40$.
8 Therefore — — $x = \frac{40}{6} = 6\frac{2}{3}$

So that there wanted $6\frac{2}{3}$ of Water to fill it: To which add Wine 20 Gallons, it is $26\frac{2}{3}$ Gallons the Contents.

Note. Since the Caſk full at 6*s.* per Gallon is worth 160*s.* it is evident that the Number of Gallons it contains is $\frac{160}{6} = 26\frac{2}{3}$. EDIT.

PROOF.

PROOF.

	£.	s.
20 Gallons of Wine, at 8s. =	8	0
And $26\frac{2}{3}$ of mixt, at 6s. =	8	0

*PROBLEM XXXIV.

Vitruvius, (Lib. ix. Chap. 3.) *informs us, That King* Hiero *being obliged by Vow to make a Preſent of a Crown of pure Gold, weighing* 100*lb. gave Orders for ſuch an one to be made; but being told that the Goldſmith had ſecreted Part of the Gold, and put to the Crown the ſame Weight of Silver, he ſent for the famous* Archimedes *of* Syracuſe, *to whom he recommended the Diſcovery of the Fraud: It is demanded how* Archimedes *diſcovered the Cheat, and how many Pounds of Silver the Goldſmith had put into the Crown?*

☞ Since it is proved by Experiment that a Maſs of pure Gold will poſſeſs leſs Space than a Quantity of Silver of the ſame Weight, it will be eaſy to conceive that a mixt Maſs of Silver will poſſeſs, or take up a Space between them. *Archimedes* therefore cauſed two Maſſes to be made of equal Weight with the Crown, one all of pure Gold, and the other all Silver; then having a Veſſel filled to the Brim with Water, he cauſed the Crown to be immerged, carefully reſerving the Water which flowed over. And thus he did with the Maſs of Gold and Silver, reſerving each Time the Water which flowed over the Veſſel; by which Means he very exactly told *Hiero* how much Gold was ſecreted.

Now let us ſuppoſe, that by immerging the Maſs of Gold only, there was emitted 60*lb.* of Water, and by the Maſs of Silver 90*lb.* and by the Crown 64*lb.* Then,

Numerical Solution.

| 1 | For the Quantity of Silver in the Crown put x,
| 2 | Then muſt the Gold — — $100 - x$.

3 Now if 100lb. of Gold be equal in Bulk to 60lb. of Water,

$100 - x$ is $\frac{6000 - 60x}{100}$

4 Again, If 100lb. of Silver be of equal Bulk with 90lb. of Water,

x is $\frac{90x}{100}$

5 The Sum of the two is $\frac{6000 + 30x}{100} = \frac{600 + 3x}{10}$

6 This ought then to be equal in Magnitude to the Pounds of Water emitted by the Crown, *viz.* 64.

Whence, $\frac{600 + 3x}{10} = 64$.

7 This reduced, — $600 + 3x = 640$,

8 That is — $3x = 640 - 600$,

9 That is, — — $3x = 40$,

10 Therefore, — $x = \frac{40}{3} = 13\frac{1}{3}$.

So that the Goldſmith had mixt $13\frac{1}{3}$lb. of Silver in the Crown.

Note 1. But there may be an infinite Number of Anſwers produced, according to the Variation of the Queſtion, and the Crown will be more or leſs adulterated, according to the Proportion of the Water emitted by the Maſs of Gold and the mixt Maſs; for the leſs their Difference, the leſs the Adulteration.

Note 2. That this and ſuch like Queſtions may be done by knowing the ſpecific Gravity of each Body; that is, weighed ſeparately in Air and Water, and their Proportions will hold good in the ſame Manner as above, and is modernly practiſed.

Note 3. By the Editor. *The above Solution is grounded upon a Suppoſition that a mixed Maſs of Gold and Silver takes up a Space equal to the Sum of the Spaces taken up by equal Quantities of thoſe Metals when ſeparate; but this Hypotheſis, I apprehend, has not been proved by Experiments.*

PROBLEM XXXV.

What Number is that whoſe $\frac{1}{2}$, $\frac{1}{3}$, *and* $\frac{1}{4}$ *exceed the Whole by* 240?

Numerical Solution.

1 | For the Number put x,
2 | Then will its $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, be $\frac{x}{2}$, $\frac{x}{3}$, $\frac{x}{4}$.
3 | Theſe reduced to a C. D. will be $\frac{12x + 8x + 6x}{24}$,
or rather, $\frac{26x}{24} = \frac{13x}{12}$.
4 | Whence (Q.) this Equation, $\frac{13x}{12} = x + 240$.
5 | This reduced, —— $13x = 12x + 2880$,
6 | And φ $12x$, —— $13x - 12x = 2880$.
7 | That is, —— $x = 2880$, the Number.

PROBLEM XXXVI.

There is a Rod of Iron a Yard long, at the Ends of which hang 2 Weights, viz. *one of* 15, *and the other of* 1 lb. *Weight: I demand the Point of the Rod where theſe two Weights will hang in Balance?*

Numerical Solution.

1 | For the Diſtance of the leſs Weight to the Point put x,
2 | Then will the Remainder be the Diſtance of the Greater, *viz.* $36 - x$ Inches.
3 | Then will the Proportion be,
As $x : 36 - x :: 15 : 1$.
4 | Multiplying Means and Ext. $1x = 540 - 15x$,
5 | Then $\varphi - 15x$, —— —— $16x = 540$.
6 | Therefore $x = \frac{540}{16} = 33\frac{3}{4}$. Leſs Weight.

| 7 |

7 And by 2*d* Step, $36 - x = 2\frac{1}{4}$. Greater.
So that the Point from the ſmall Weight muſt be $33\frac{3}{4}$ Inches, and the Diſtance of the 15*lb.* Weight 15 Times leſs, *viz.* $2\frac{1}{4}$ from the End.

PROBLEM XXXVII.

Suppoſe a Rod of Iron to be divided into 150 *equal Parts, and at the firſt Part or Diviſion hangs a* 4 *lb. Weight, and on the laſt Diviſion, or other End, a Weight of* 4 *Score and* 16 *Pounds: I demand the Point of the Rod where thoſe two Weights will be in* Equilibrio; *or, which is the ſame, what Diviſion of the Rod will be a Balance to both the Weights?*

Numerical Solution.

1 For the Diſtance of the Point from the greater Weight put x;
2 Then will the Diſtance from the leſs be $150 - x$.
3 Then ——— As $x : 150 - x :: 4 : 96$.
4 By multiplying Means and Ex. $96x = 600 - 4x$.
5 Then $\varphi - 4x$, —— —— $100x = 600$.
6 Therefore, $x = \frac{600}{100} = 6$, the Diſtance of the Point from greater Weight.
7 And by 2*d* Step, $150 - x = 144$, its Diſtance from leſs Weight.

☞ From hence alſo if a Rod be divided into any Number of Parts, and one Weight be given, and the Point given in the Rod, the other Weight is eaſily found by Proportion, thus: As the Diſtance of one End is to the given Weight, ſo is the other Diſtance from the End to the required Weight. Thus, in the *Problem* before us, let the Weight 6 be given, and the Diviſion 144 on which it is placed, and the whole Rod 150, as before, to find what Weight will balance it.

PROBLEM XXXVIII.

There is a Rod divided into 150 *equal Parts, on which hangs at one End a* 4*lb. Weight, and the Rod being placed or laid acroſs any Thing at the* 144*th Diviſion, I demand what Weight at the other End will be able to balance the* 4*lb Weight, to keep the Rod in* Equilibrio?

This is done by the Help of the Rule of Proportion, either direct or inverſe. For only obſerving the Diſtance the Point of Balance is from the given Weight, (which here is 144, and the Remainder 6) the Proportion is, As 6, the Remainder of Diviſions from the leſs Weight, is to the Weight itſelf, ſo is the Diſtance of the leſs Weight from the Point of Balance to the greater Weight, &c. Thus,

As 6 : 4 : : 144 : 96. Or,
As 6 : 144 : : 4 : 96, &c.

☞ By this Method may be proved, whether the *Steel-yard*, or any Beam or Pair of Scales belonging be good: For notwithſtanding the vulgar Notion of Beams and Scales being true, becauſe the Brachium hangs in Balance, it is evident, that in weighing large Quantities the Buyer may be ſufficiently cheated, or the Seller may ignorantly cheat himſelf, and *that* more or leſs, in Proportion according to the Make of the Beam.

Tyr. This is a Sort of a Paradox to me at preſent!

Nov. And to me likewiſe; I wiſh, therefore, *Philomathes*, you would explain it a little to me: For if it be ſo, how am I to chooſe a good Beam, or depend upon others to know whether they be true or falſe; for it is not to be ſuppoſed every one can prove it by Figures?

Phi. I grant it; but you may ſoon ſee the Truth of this by ſeveral Experiments*. However, I will tell you thus much, that in chooſing your Beam, mind not altogether its hanging in Balance; but more particularly examine whether the Arms whereon the Scales hang be equi-diſtant from the Center; for ſhould they not, you may depend it is not an honeſt Balance: And whenever you ſuſpect any Pair of Scales, you may ſatisfy yourſelf by this vulgar Experiment only; change the Weights and Commodity to the contrary Scale, and if the Weight be as before, it is right, otherwiſe falſe.

---

* See Dr. *Deſagulier's Experimental Philoſophy*, Vol. 1. Plate 7, 8, 9, and 10.

Nov. We are obliged to you for this eaſy Explanation.

Phi. I am not treating of *Mechanics*, it is true; but I have ſaid ſomething the more upon it, becauſe it is more uſeful and neceſſary in Buſineſs than every common Queſtion; and indeed, ſuch Perſons as deal in valuable Commodities in large Quantities, ſhould be careful to examine Things of this Nature. But come, we will proceed to

PROBLEM XXXIX.

A Tradeſman began the World with a certain Sum of Money, with which he bought a Stock of Goods, but by Misfortunes in Trade, he loſt the firſt Year one Half the Value of his Stock, and 10 *Shillings over; and alſo the ſecond Year he loſt Half his Stock, and* 10 *Shillings over; and thus he went on for* 5 *Years, loſing Half the Value of the preceding Year's Stock, and* 10 *Shillings over: Now at the End of the fifth Year he left off Trade, and his Stock was worth but* 50*l.* 10*s. I demand the Value of his Stock at firſt?*

Numerical Solution.

1 For the Value of his Stock at firſt in Pounds put x,

2 Then he loſt the firſt Year — $\frac{x}{2}+\frac{1}{2}$ *.

3 And had left — — $\frac{x}{2}-\frac{1}{2}$, *viz.* $\frac{x-1}{2}$.

---

* *Note*, As 10 Shillings is Half of a £. the $\frac{1}{2}$ in the 2*d* Step repreſents 10 Shillings, and ſaves a great deal of Trouble.

4 Then had he loſt juſt ½ this, the 2d Year it would be — — — $\frac{x-1}{4}$.

5 But he loſt 10 Shillings more, therefore he had left — — — $\frac{x-1}{4} - \frac{1}{2}$.

6 This reduced, firſt × the Denominator 4, then × 2, is — — $\frac{2x-2-4}{8}$ or, $\frac{2x-6}{8}$.

7 This abbreviated, is the 2d Year's Stock left $\frac{x-3}{4}$.

8 Then by loſing Half + 10s. had left the 3d Year, — — — $\frac{x-3}{8} - \frac{1}{2}$.

9 This reduced, is $\frac{2x-6-8}{16}$, or, $\frac{2x-14}{16}$, or, $\frac{x-7}{8}$.

10 The Half of this, leſs 10 Shillings is left the 4th Year, *viz.* — $\frac{x-7}{16} - \frac{1}{2}$. Or,

11 This reduced, is $\frac{2x-14-16}{32}$ or, $\frac{2x-30}{32}$. or, $\frac{x-15}{16}$.

12 Then had he left half this, leſs 10 Shillings the 5th Year, *viz.* $\frac{x-15}{32} - \frac{1}{2}$.

13 This reduced, is $\frac{2x-30-32}{64}$ or, $\frac{2x-62}{64}$, or, $\frac{x-31}{32}$.

14 Now this (Q.) is equal £. 50. 10s. or, 50½. Whence, — $\frac{x-31}{32} = 50\frac{1}{2}$.

15 This reduced, by × Denominator 32, $x - 31 = 1600\frac{32}{2} = 1600\frac{16}{1} = 1616$,

16 | Then $\varphi - 31$, $x = 1616 + 31$
17 | Therefore, $x =$ £. 1647, his *1ſt* Stock, which you may prove at your Leiſure.

PROBLEM XL.

What Number is that which if added ſeverally to 3, 19, *and* 51, *will make them* 3 *Proportionals?*

Numerical Solution.

1 | For the Number put x,
2 | Then by adding this to each Number, they are $x + 3$, $x + 19$, $x + 51$.
3 | Whence, by the Rule of Proportion,
| As $x + 3 : x + 19 :: x + 19 : x + 51$.
4 | But multiplying *Means* and *Extremes* you have
| $xx + 54x + 153 = xx + 38x + 361$
5 | Then by cancelling xx on both Sides,
| $54x + 153 = 38x + 361$.
6 | Then φ $38x$ and 153, it is
| $54x - 38x = 361 - 153$,
7 | That is, $16x = 208$.
8 | Therefore $x = \frac{208}{16} = 13$ *Anſ.*

PROOF.

Numbers	3	19	and	51
Add	13	13		13
	16	32		64

For $16 \times 64 = 32 \times 32$.

Do theſe Operations appear plain to you, *Novitius?*

Nov. Very plain, Sir; I think I underſtand them all very well.

Tyr. I wiſh I could ſay ſo for my Part, for I muſt own at preſent I do not.

Phi. It is not to be expected ſo young a Learner as you, *Tyrunculus*, ſhould be Maſter of theſe Things at once: If you underſtand the Work by reading it, that is ſufficient at preſent, and in going through the *Pro-*

blems once more, you will, no Doubt, underſtand them; and I think I have given you a Variety of Examples enough.

Nov. I beg you would work a Queſtion or two in plain *Trigonometry* before you conclude, if *ſimple Equations* will perform them.

Phi. Yes, there are many to be performed by *ſimple Equations* only: But really I have ſcarce Room to grant your Deſire, having already added ten *Problems* more than I intended. However, to oblige you I will; but then I ſhall work them *literally*, for it will be too tedious to do them *numerically*.

PROBLEM XLI.

Suppoſe a Pole to ſtand upon a Horizontal Plane, 75 *Feet clear from the Ground (or* c*); what Height from the Ground muſt it be cut or ſawn off at, ſo that the Top of it may fall upon a Point* (b) 55 *Feet from the Bottom of the Pole to the ſaid Point on the Ground?*

Literal Solution.

1 For the Height ſought put x,

2 The Square of which is xx,

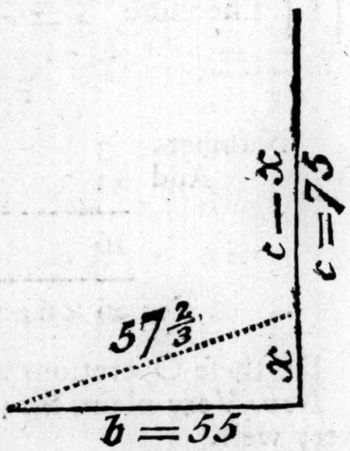
3 Then is the Square of b ———— bb,

4 The Sum of their Squares $xx + bb$.

5 The Remainder to 75 is — $c - x$,

6 The Square of this is $cc - 2xc + xx$.

7 Whence this Equation, $xx + bb = cc - 2xc + xx$.



8 Cancel xx on both Sides, $bb = cc - 2xc$.

9 Then φ — $2xc$, it is —— $2xc + bb = cc$.

10 Then φ bb, ——— $2xc = cc - bb$.

11 Then muſt $x = \dfrac{cc - bb}{2c}$, that is, $x = 75 \times 75 - 55 \times 55$, divided by 75×2, which is $= 17\frac{1}{3}$ Feet. *Anſ.*

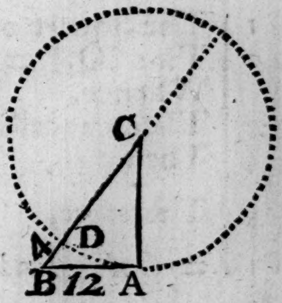
Note, This Problem may be conſtructed geometrically, in the ſame manner as Problem 43. EDIT.

PROBLEM XLII.

In the Triangle ABC *is given the Baſe* AB = 12, *and the Segment, or Part of the Hypothenuſe* BC, *viz.* BD = 4. *Required the Sides* AC *and* BC?

Literal Solution.

Let $a = 12 = AB$, and $b = 4 = BD$; and let $x = AC$. Now as AC and CD are equal, BC muſt be $= b + x$. Then by the 47*th* of I. *Euclid*, $BC^2 = AC^2 + AB^2$; but $BC^2 =$ the Square of $b + x$, *viz.* $bb + 2bx + xx$, and $aa = AB^2$.



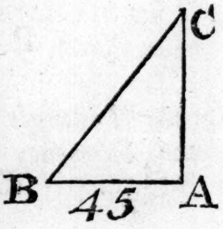
1 The Square of $b + x$ is $bb + 2bx + xx$.
2 This (Q.) is = Square of x and a.
Whence, —— $bb + 2bx + xx = xx + aa$.
3 By cancelling xx on both Sides, $bb + 2bx = aa$.
4 Then φ bb, it will be $2bx = aa - bb$.
5 Therefore, $x = \frac{aa - bb}{2b} = \frac{12 \times 12 - 4 \times 4}{2 \times 4} = 16$
So that the Side CA or $CD = x = 16$. And $CD + BD = 16 + 4 = 20 =$ Hypothenuſe CB.

PROBLEM XLIII.

There is a rectangled Triangle ABC, *whoſe Baſe* AB = 45, *and the Sum of the Hypothenuſe and Cathetus* AC + BC = 135: *It is required to find the Sides* AC *and* BC *ſeparately?*

Literal Solution.

Let $d = 135 = AC + BC$, and let $b = 45$, or BA; and put x for CA; then $CB = d - x$. And ſeeing the Angle CAB is a right one, we have (by the 47*th* of 1 *Euc.*) $BC^2 = CA^2 + BA^2$; but $BC^2 =$ the Square of $d - x$, *viz.* $dd - 2dx + xx$. And $CA = xx$; and $BA = bb$. Therefore



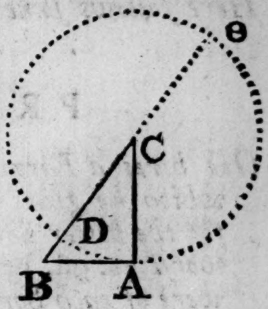
1 The Square of $d - x$ is $dd - 2dx + xx$.
2 This (Q.) = to the Square of x and b.
Whence, $dd - 2dx + xx = xx + bb$.
3 Then cancelling xx, $dd - 2dx = bb$.
4 Then $\varphi\ bb$, ——— $dd - bb = 2dx$.
5 Therefore, $x = \frac{dd - bb}{2d}$ or $\frac{d}{2} - \frac{bb}{2d} = 60$
$= CA$. And $135 - 60 = 75 = BC$.

O R,

Let $x = BC$, and then will $AC = d - x$; and therefore (by the 47*th* of 1 *Euc.*) $BC^2 = AC^2 + AB^2$. Therefore, as $b = BA$, as above, it will be

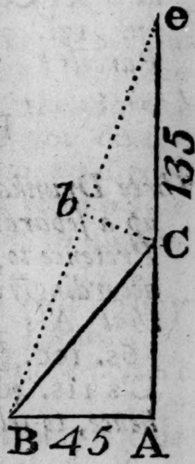
1 $xx = dd - 2dx + xx + bb$.
2 $dd - 2dx + bb = 0$
3 $2dx = dd + bb$,
4 $x = \frac{d}{2} + \frac{bb}{2d} = \frac{135}{2} + \frac{45 \times 45}{2 \times 135} = 75$ the Hypothenuſe BC. And $135 - 75 = 60 = AC$, as before.

Note, By the 36*th* of 3 *Euclid*, you may find the Sides BC and AC thus: Deſcribe a Circle, making the Perpendicular the Radius; then is the Rectangle Be into the Segment $BD =$ the Square of AB; therefore BD is $=$ the Square of AB divided by $Be = BC + AC = 135$; that is, $BD = \frac{45 \times 45}{135} = 15$; and therefore (by the Figure and the preceding Work) $BC + AC - BD \div 2$; that is $\frac{135 - 15}{2} = 60$ AC, and $60 + 15 = 75$ BC.



Geometrically.

Firſt, From any Scale of equal Parts make the Line $AB = 45$, and at right Angles to it draw $Ae = 135$, the Sum of the Sides AC and BC. Then join Be with a right Line from e to B, and divide this in the Middle at b; then erect a Perpendicular from b to cut the Line Ae, which will be in the Point C; then having drawn the Line BC, the Triangle is compleated: And if you meaſure the Sides upon the ſame Scale, you will find $BC = 75$, and $AC = 60$, as above: For the Line bc being Perpendicular to Be, and cutting it into two equal Parts; the Triangle BCe is *Iſoſcles*, by the 5*th* of 1 *Euc.* Therefore conſequently $AC + BC = AC + Ce$. Q. E. D.



And thus, *Novitius*, I have done all that is in my Power to ſerve you and *Tyrunculus*; and I ſhall leave 7 *Problems* more, without their Operation, (to make up the Number 50) for your Practice; and I deſire you would aſſiſt *Tyrunculus* in them, as I have aſſiſted you in the others.

SECT.

SECT. II.

Here follows ſome more PROBLEMS, *to exerciſe the young* ALGEBRAIST.

PROBLEM XLIV.

One hires a Farm, containing 125 *Acres of Ground, for which he gives* 38l. 5s. *the Land conſiſts of two Sorts; for the better Sort he gave* 7s. 6d. per *Acre, and for the worſt* 3s. 9d. per *Acre: I demand how many Acres there were of each Sort.*

PROBLEM XLV.

One lets out 60*l. in* 2 *Parcels,* viz. *one at* 5, *and the other at* 6 per Cent. *which in* 13 *Years ſimple Intereſt wanted but* 19l. 7s. 6d. *of the Principal: I demand the Parcels?*

PROBLEM XLVI.

Three Drunkards, A, B, *and* C, *having each of them run up a ſeparate Score at an Alehouſe, agreed to go (under Pretence to drink) and rub all out; which was done accordingly; but the Landlord remembered very well, that* A's *and* B's *Reckoning added together made* 6s. 10d. $\frac{1}{2}$, *and* B's *and* C's 13s. 3d. $\frac{1}{4}$, *and* A's *and* C's 11s. 5d. $\frac{3}{4}$: *He therefore craves your Aſſiſtance from hence, to tell him each Man's diſtinct Score?*

PROBLEM XLVII.

There are two Numbers whoſe Sum is 517, *and the Quotient of the greater by the leſs is juſt* 1000: *I demand the Numbers?*

PROBLEM XLVIII.

What Number is that, which, if added to 23, 209, *and* 561, *will make them* 3 *Proportionals?*

PROBLEM XLIX.

What Number is that whoſe $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, *and* $\frac{1}{12}$, *exceeds itſelf by* 1?

PROBLEM L.

A Perſon dying left in Caſh 410*l.* 10*s. to his* 4 *Sons,* A, B, C, *and* D, *in ſuch Manner that if* A *had* 4*l.* 10*s. more, and* B 4*l.* 10*s. leſs, and* C's *were multiplied by* 4*l.* 10*s. and* D's *divided by* 4*l.* 10*s. it would produce one and the ſame Sum of Money: What was the Portion of each?*

Nov. But pray why do you not inſert their Anſwers?

Tyr. It would be ſome Help, I think to the young Practitioner.

Phi. It would be indulging him you might ſay indeed, but I cannot ſee why the Queſtions are at all the harder to be done, without it be to ſuch who from the Anſwers often gueſs out the Numbers; therefore I chooſe rather to omit the Anſwers: For if you do the Work right, it will prove itſelf; and to a diligent Learner it is all the ſame as if the Anſwers were before him; and I am ſure it is a proper Exerciſe to qualify you for more difficult Things of this Sort.

Nov. You may depend upon it, Sir, I will do my Endeavour to find their Anſwers in a ſhort Time.

Tyr. So will I, as ſoon as I am a little more perfect in the foregoing *Problems.*

Phi. You are right, *Tyrunculus,* for from a true Knowledge of them you will ſoon diſcover theſe alſo.

Nov. We are highly obliged to you, *Philomathes,* for theſe Favours.——Come, *Tyrunculus,* do you think of going?

Tyr. When you pleaſe, Sir.

Nov. Dear *Philomathes,* in accepting my hearty Thanks you will yet more oblige your humble Servant,

Tyr. Pray receive mine alſo.

Phi. I do; and you are not only welcome to theſe ſmall Inſtructions, but I ſhall always be ready to ſerve you: Only let me perſuade you (as far as Things of more Moment will allow of) to aſſiſt one another; for it is poſſible I may (by and by) inſtruct you in ſomething of *Quadratic Equations*, becauſe it is a Pleaſure to me to ſee you delight thus in Figures. Thoſe that have no Taſte for this Sort of Learning indeed, are ignorant of the Satisfaction that it leaves; for what can be a greater Satisfaction to the Mind than *Certainty* itſelf, built upon the Foundation of unerring Principles? This made a noted Author ſay, that "*Algebra*, like "*Logic*, gives us a juſt Idea of the Nature of Things, "ſhews us the true Way of Reaſoning, elevates the "Mind to a proper Degree, and will not ſuffer it to "dwell upon mean and baſe Trifles."

And I could heartily wiſh that more of the growing Youth of this Age (eſpecially ſuch as can afford it) would (with you) give their Minds to the Study of ſome of the *Mathematical Sciences*, they being not only uſeful, but very diverting, and would certainly tend much more to their own private Good, and *that* of others, than the conſtant Peruſal of ſuch Books which daily vitiate the Mind, and corrupt the Morals. Thus we read, "*Xenophon* commended the *Perſians* for their "careful and prudent Education of their Children, who "made them ſtudy only ſuch Authors as treated of "Learning and Morality; but would not ſuffer them "to effeminate themſelves with idle and amorous Tales, "knowing well, and wiſely diſcerning, *that there needed* "*no Weight to be added to the* Bias *of corrupt Nature*."

AN

APPENDIX,

CONTAINING

Some neceſſary INSTRUCTIONS

In the RUDIMENTS of

QUADRATIC EQUATIONS:

VIZ.

I. INVOLUTION, or the Method of raiſing POWERS or involving QUANTITIES.

II. The RESOLUTION of a SQUARE raiſed from a BINOMIAL, and how to compleat the SQUARE.

III. Of EVOLUTION or extracting ROOTS.

APPENDIX, &c.

In a DIALOGUE between

PHILOMATHES and *NOVITIUS*,

CONTAINING

Some neceſſary INSTRUCTIONS

IN

QUADRATIC EQUATIONS.

PHILOMATHES *calls upon* NOVITIUS *to know what Improvement he has made in* SIMPLE EQUATIONS.

@ *The Sign of* INVOLUTION.
μυ *The Sign of* EVOLUTION.
√ *The Sign of* SURD *or* IRRATIONALITY.

NOVITIUS, your Servant.

Nov. Sir, I am heartily glad to ſee you.

Phi. You remember I promiſed to give you ſome Notion of *Quadratics*, which I intended to have done before, but that Buſineſs of greater Moment has continually called for my Attention. And though I am now come according to Promiſe; yet my Viſit will be but ſhort: And before I begin with you let me know whether you are pretty perfect in what I have ſhewn you before.

Nov. That I aſſure you I am.

Phi. We will proceed to the Point in Hand then.

Nov. Pray what does the Knowledge of *Quadratics* depend upon?

Phi. The Knowledge of *Quadratic Equations* depends upon theſe 4 Things.

1ſt. The Method of raiſing Powers from a ſingle Quantity.

2dly. The Reſolution of a Square raiſed from a *Binomial* or *Reſidual*; (i. e. of the three Members which ſuch a Square conſiſts of.)

3dly. How to complete the Square when two Members only are given. And

4thly. The Way or Method of extracting the Roots.

The firſt two of theſe are comprehended under the Name of *Involution*, or the Method of involving Quantities from any given Root.

Of raiſing Powers from a ſingle Quantity.

RULE.

Multiply the given Quantity into itſelf you have the Square, to which join the ſaid Quantity you have the Cube or third Power, &c. &c.

EXAMPLE I.

Let x the Root be involved to the 2, 3, 4 and 5*th* Power.

x Root
xx Square or 2*d* Power.
xxx or x^3 Cube or 3*d* Power.
$xxxx$ or x^4 Biquadrate or 4*th* Power.
$xxxxx$ or x^5 Surſolid or 5*th* Power.

Nov. This is ſo plain, more Examples are needleſs: But what do you mean by a *Binomial* and *Reſidual* Root?

Phi. A *Binomial* is a compound Quantity conſiſting of two Parts, as $x + b$, or $x + \frac{b}{4}$, connected together by the Signs +; and $x - b$, or $x - \frac{b}{4}$, connected by the Sign —. Now theſe two Parts multiplied by themſelves (that is ſquared) will always produce 3 Members, the firſt and laſt of which will be perfect Squares, and will always be *affirmative*; and the middle Part or Member is made by the double Rectangle of the Parts of which the *Binomial* is compoſed, and this middle Part will be ſometimes *Affirmative*, and ſometimes *Negative*: *Affirmative* when both are *Affirmative* or *Negative*, as $x+b$ or $-x-b$; and *Negative* when one of the Parts is *Negative*, as $x-b$*; do you underſtand me?

Nov. Yes, very well, except it be the double Rectangle you talk of.

Phi. This I ſhall ſatisfy you about preſently, under Obſervation the Firſt: In the mean Time, we will give you ſome Examples of *the Reſolution of a Square raiſed from a* Binomial *or* Reſidual, *or of the three Parts which ſuch a Square conſiſts of*; *and how to complete the Square when two Members only are given.*

EXAMPLE 2.

Let $x+b$ a *Binomial* be raiſed or involved to the third Power.

$$\begin{array}{llll}
x+b & & & \\
x+b & & & \\
\hline
xx+xb & & & \\
\quad\;\; xb+bb & & & \\
\hline
xx+2xb+bb & \text{the Square.} & \text{This} \times \overline{x+b} & \\
x+b & & & \\
\hline
xxx+2xxb+xbb & & & \\
\qquad\quad xxb+2xbb+bbb & & & \\
\hline
xxx+3xxb+3xbb+bbb & \text{Cube.} & &
\end{array}$$

---

* *Note,* A *Binomial* is called a *Reſidual* Root when one Part is Negative, as $x-b$.

EXAMPLE 3.

Let $x - b$ a *Reſidual* be raiſed or involved to the third Power.

$$\begin{array}{l} x - b \\ x - b \\ \hline xx - xb \\ \quad\;\; - xb + bb \\ \hline xx - 2xb + bb \text{ Square.} \quad \text{This} \times \overline{x - b} \\ \;\; x - b \\ \hline xxx - 2xxb + xbb \\ \quad\;\; - \;\; xxb + 2xbb - bbb \\ \hline xxx - 3xxb + 3xbb - bbb \text{ Cube.} \end{array}$$

Nov. I underſtand you very well; and I alſo perceive that in the *Binomial* the Anſwer is affirmative in all the Quantities; but in the *Reſidual* they alternately change.

Phi. You ſay right; and you ſee that it is nothing but common Multiplication; and if there were Fractions the Work is the ſame.

EXAMPLE 4.

Thus, $\frac{x}{2}$ ſquared is $\frac{xx}{4}$, for only multiply the Denominators and the Numerators together as in Multiplication of Algebraic Fractions. So alſo $\frac{x}{4}$ ſquared is $\frac{xx}{16}$; and the Square of $x + \frac{a}{2}$ a *Binomial*, is $xx + \frac{2ax}{2} + \frac{aa}{4}$, or $xx + ax + \frac{aa}{4}$; and laſtly, the Square of $x - \frac{a}{2}$ will be $xx - \frac{2ax}{2} + \frac{aa}{4}$, or $xx - ax + \frac{aa}{x}$. For 2 in the middle Term being common to the Numerator and Denominator, I expunge it, and take the Numerator ax only.

Nov. I thank you kindly, Sir, for your demonſtrating it ſo plainly.

Phi. I preſume then, as you know how to involve any Root, you alſo know the third Thing, that is, how to complete the Square.

Nov. That I do not, nor do I know altogether at preſent what you mean.

Phi. I own, as I ſaid before, that it is not ſo eaſily known by a Learner; therefore I readily excuſe it, becauſe Authors in general have taken *no* Notice of this, though ſo neceſſary.

Of compleating the Square when but two Members are given.

I have already told you, that any compound Quantity, whether *Binomial* or *Reſidual*, when ſquared will conſiſt of three Members; the Middle of which will be ſometimes *affirmative*, and ſometimes *negative*: I alſo told you, that when they are both perfect Squares you may eaſily know it, as $xx + bb$, or $xx + 16$; to compleat the Square of which will eaſily appear as follows.

OBSERVATION I.

When any compound Quantity as $xx + bb$ wants to be compleated, that is, wants the middle Part or Member, (for remember I told you it conſiſts of 3) then take the Root of each Part, *viz.* x and b, and multiply them together, which is xb or $-xb$; this is what we call the Rectangle, the double of which is $2xb$ or $-2xb$, either of theſe put between the other two will compleat the Square.

Nov. This is eaſy indeed if I take it right; for ſuppoſe $xx + bb$ were to be compleated, the Root of xx is x, the Root of bb is b; now $x \times b = xb$ the ſingle Rectangle, therefore $2xb$ is the double Rectangle, which, placed between the other two Members will be $xx + 2xb + bb$, or $xx - 2xb + bb$. But how am I to proceed when there are Co-efficients?

Phi. The very ſame: For ſuppoſe the Root $8x - 2$ were to be ſquared or compleated, here $8x \times 8x = 64xx$ the firſt Term, and $- 2 \times - 2 = 4$ the third Term. Now $8x \times - 2 = - 16x$ the ſingle Rectangle, therefore $- 32x$ is the double Rectangle or middle Term; ſo the Square compleated is $64xx - 32x + 4$.

Numerical Demonſtration.

Suppoſe they were in Numbers only, you will ſee the middle Term is always made up with the double Rectangle of the Parts. For let any Number (ſuppoſe 16) be divided into any two Parts, as 12 and 4: To compleat the Square $4 \times 12 = 48$, which doubled, is 96 the middle Term, ſo is the Square compleated, *viz.* $144 + 96 + 16$; and if you make the *Binomial* $x + b = 12 + 4$, you will in courſe have $xx + 2xb + bb = 144 + 96 + 16$.

Nov. I like this very well; but pray how do you compleat the Square, when, of the two given Members, only the firſt is a Square, and the other any Quantity or Number propoſed at Random.

Phi. To be ſure this cannot be done in many Caſes,* by taking the double Rectangle of the Parts as before directed, becauſe the Parts conſiſt not of two pure or perfect Squares. But I will put you into another Way of compleating the Square, which was the third Thing propoſed, namely,

OBSERVATION 2.

When any two Quantities are propoſed, whereof the firſt is a Square, and the ſecond Member is the Root of that Square, with a Co-efficient, whether that Co-efficient be a whole Number or a Fraction, you may always find the third Member, and compleat the Square, by this general Rule.

---

* This may be always done, and eaſily too, by the Doctrine of Surds; but ſuch Caſes very ſeldom occur in the Solution of Problems. EDIT.

RULE.

Take Half the Co-efficient of the ſecond Member, and the Square thereof ſhall be the third Member, which will compleat the Square of the ſaid two given Members.

Nov. What, do you ſay this will do, though I propoſe any Quantities or Fractions at Random?

Phi. Yes, provided your firſt Member be a perfect Square, and the ſecond has the Root of that Square found in it.

Nov. Give me a few Examples.

Phi. I will.

EXAMPLE 1.

Suppoſe $xx + 8x$ were to be compleated; here, Half the Co-efficient of the ſecond Member, *viz.* 8, is 4, whoſe Square is 16, which will be the third Term: $xx + 8x$ therefore, when compleated, will be $xx + 8x + 16$, the Root of which is $x + 4$, for $\overline{x + 4} \times \overline{x + 4} = xx + 8x + 16$.

EXAMPLE 2.

Let $xx + 14x$ be compleated.

Here Half of 14 is 7, this ſquared is 49; ſo that $xx + 14x$, when compleated, is $xx + 14x + 49$.

Nov. Very eaſy indeed, and very pretty.

Phi. Notwithſtanding this you ſhall very rarely meet with it in Authors.

Nov. I know it; but pray, ſuppoſe the ſecond Member have an odd Number or Co-efficient, or ſuppoſe it to have Fractions, how then?

Phi. The very ſame as before.

EXAMPLE 3.

Let $xx + 5x$ be compleated.

Nov. I am at a Loſs at preſent indeed.

Phi. Surely not, *Novitius!* why, is not the Half of 5 expreſſed $\frac{5}{2}$?

Nov. I aſk Pardon, it is ſo, and the Square of $\frac{5}{2}$ is $\frac{25}{4}$, is it not?

Phi. You are right.

Nov. Then I perceive $xx + 5x$ when compleated, will be $xx + 5x + \frac{25}{4}$. And by the ſame Rule $xx - \frac{2x}{5}$ will be $xx - \frac{2x}{5} + \frac{1}{25}$; for the $\frac{1}{2}$ of $\frac{2}{5}$ is $\frac{1}{5}$, and the Square of $\frac{1}{5}$ is $\frac{1}{25}$. Shew me one or two literally.

Example 5.

Suppoſe $xx + bx$ be given, what will the third Member be to compleat the Square?

Here the Co-efficient of the ſecond Member is b, Half of which is $\frac{b}{2}$, which ſquared is $\frac{bb}{4}$; ſo that $xx + bx$, when compleated, is $xx + bx + \frac{bb}{4}$. See *Page* 188. *Example* 4.

Example 6.

$xx + \frac{bx}{a}$ when compleated, is $xx + \frac{bx}{a} + \frac{bb}{4aa}$. For half the Co-efficient $\frac{b}{a}$ is $\frac{b}{2a}$, the Square of which is $\frac{bb}{4aa}$. Are you ſenſible of this?

Nov. Nothing appears plainer.

Phi. Since you know ſomething of the Nature of *Involution*, and completing the Square, I will now give you a Notion of *Evolution* directly.

3. *Of* Evolution.

Evolution is the Reverſe of *Involution*, and ſhews us how to extract the Root of any given Power.

Example

EXAMPLES,

xx
x Root
$xx\ bb\ cc$
$x\ \ b\ \ c$ Root

$xxxx$
xx Root.
$xx\ bbbb.\ dddd$
$x\ \ bb\ \ dd$ Root.

OBSERV. I.

When there are ſeveral Quantities in one Power, Then conſider which of thoſe Powers are perfect or pure Squares of themſelves; for ſhould the firſt and third be ſo in any Power raiſed from a *Binominal* or *Reſidual*, extract the Root of the ſaid two Powers, and you have the ſquare Root of the whole Quantity or Power. Thus,

$$\begin{array}{l} \underline{xx + 2xb + bb} \text{ Square} \\ \quad x + b \quad \text{Root.} \end{array}$$

For the Root of xx is x, and the Root of bb is b, and theſe two connected are $x + b$, and theſe I ſuppoſe the true Root; but I find it to be ſo upon two Trials, firſt $x \times b = xb$, this doubled is $2xb$ the middle Term; alſo $\overline{x + b} \times \overline{x + b}$ gives $xx + 2xb + bb$, the Power given. Again,

$$\begin{array}{ll} x^4 + 6\,xxx + 9\,xx & \text{Square} \\ xx + 3\,x & \text{Root.} \end{array}$$

Alſo ſuppoſe $xxxx - 14\ xxbb\ cc + 49\ bbbb\ cccc$.
Then $xx - 7\ bb\ cc$ Root.

Here are two pure Powers, the Square Roots of which are xx and $7\ bb\ cc$; therefore I conclude $xx - 7\ bb\ cc$ to be the Root, becauſe the middle Member is *negative*, and the Square of Half its Co-efficient gives 49 in the third Member.

Nov. I underſtand you well; but how am I to extract the Square Root of Fractions?

Phi. After the ſame Manner. For,

OBSERV. 2.

If the firſt Member be a pure Power, and the Fraction alſo, you may conſider it as a perfect Square raiſed from a *Binominal* or *Reſidual* Root; extract therefore the Root of the Numerator for a new Numerator, and of the Denominator ſor a new Denominator.

EXAMPLES.

Let $xx + 3x + \frac{9}{4}$ be the Square
$x + \frac{3}{2}$ Root.

For the Root of $\frac{9}{4}$ is $\frac{3}{2}$; and $\overline{x + \frac{3}{2}} \times \overline{x + \frac{3}{2}} = xx + 3x + \frac{9}{4}$. Again,

Let $xx + 3bx + \frac{9}{4}bb$ be the Square,
then will $x + \frac{3}{2}b$ be the Root. Again,

Let $xx - \frac{bx}{a} + \frac{bb}{4aa}$ be the given Square
$x + \frac{b}{2a}$ Root.

See *Example* 6. in *Involution.* Alſo,

Let $xx - dx + \frac{1}{4}dd$ be the Square
$x - \frac{1}{2}d$ Root.

Do you underſtand it?

Tyr. Yes, very well, except in one Thing, and that ſeems very odd to me.

Phi. What is that?

Nov. Why, I perceive, the Root of the Fractions is larger than the Fraction itſelf.

Phi. Not in every Reſpect neither; for $\frac{1}{4}$ of dd may be more than $\frac{1}{2}d$; but I ſuppoſe you wonder that the

Square Root of $\frac{1}{4}$ ſhould be $\frac{1}{2}$, which is more than $\frac{1}{4}$ itſelf.

Nov. I do ſo.

Phi. That the Root of every proper Fraction is greater than the Square itſelf; you may ſee the Reaſon of this, in *Dialogue* 3. *Sect.* 3. *Note* 1. *and Note the* 3*d. Sect.* 4. *of the ſame Dialogue.*

Nov. But I wiſh you would demonſtrate it.

Phi. You aſk Things indeed foreign to the Purpoſe; however I am ready to oblige you in every Thing that may be ſerviceable: I ſhall therefore explain it by Decimal Fractions; and you will ſee it at once: Now I ſuppoſe you know in Decimals, 25 is $\frac{1}{4}$, 50 is $\frac{1}{2}$, and 75 is $\frac{3}{4}$ of any Thing.

Nov. Yes, very well, for 25 is $\frac{1}{4}$ of 100, 50 is $\frac{1}{2}$ of 100, and 75 is $\frac{3}{4}$ of 100, their reſpective Denominators.

Phi. Right; obſerve then, I only ſet down 25 as it ſtands in whole Numbers, and find the Root thereof 5, which is 5 Times leſs in the Root than the Square. I alſo in $\frac{1}{4}$ ſet down 25 with a Dot or Prick before it, thus .25, and the Square Root is ſtill 5; but I put a Prick alſo before the .5, *that* being a Decimal alſo: Now .5 you know is $\frac{1}{2}$: By this you ſee that the Square Roots of proper Fractions encreaſe in Value or Quantity in Proportion to the Decreaſe of Roots of whole Numbers.

Nov. I ſee it plainly, and I heartily thank you; but how ſhall I know when a Square is not perfect, and how am I to act in ſuch a Caſe.

Phi. That will diſcover itſelf by the foregoing Rule; that is, if there be not two pure Squares; or if the double Rectangle under the Squares make not the middle Term: in ſuch Caſes as theſe, you only put this Sign ($\surd$) before it, to ſhew it is a ſurd Quantity. For this Sign is called the *Radical Sign*, or the Sign of *Irrationality*. Thus,

The ſquare Root of xb is $\surd\, xb$; of $xx + dd$ is $\sqrt{xx + dd}$.

Now you ſee it is plain $xx + dd$ is a Surd, or a Quantity that is not a perfect Square; for the Square

3

of xx is x, and the Square of dd is d; theſe connected are $x + d$; but $\overline{x + d} \times \overline{x + d} = xx + 2xd + dd$; conſequently, $xx + dd$ is a ſurd Quantity.

So alſo the Square Root of $xx + 2xb - bb$ is $\sqrt{xx + 2xb - bb}$, becauſe the firſt is *affirmative*, and the third *negative*.

Again, The ſquare Root of $xx + 5xb + bb$ is expreſſed $\sqrt{xx + 5xb + bb}$; becauſe the middle Quantity is not juſt the Double of the Product of x and b.

And now, *Novitius*, I will give you a Table of the Powers, and ſhew you the Manner of involving them more plainly; and alſo more of the Nature of Inveſtigation by Way of Exerciſe.

A TABLE of Uſe in CONVERGING SERIES, *&c.* ſhewing by Inſpection only, The ROOTS and POWERS of ſuch ROOTS to the Twelfth ROOT; by which any higher POWERS may be found.

Root, or 1ſt Power	$x = 1$	$x = 2$	$x = 3$	$x = 4$	$x = 5$	$x = 6$	$x = 7$	$x = 8$	$x = 9$
Square, or 2d Power	$x^2 = 1$	4	9	16	25	36	49	64	81
Cube, or 3d Power	$x^3 = 1$	8	27	64	125	216	343	512	729
Biquadrate, or 4th Power	$x^4 = 1$	16	81	256	625	1296	2401	4096	6561
Surſolid, or 5th Power	$x^5 = 1$	32	243	1024	3125	7776	16807	76328	59049
Square-cubed, or 6th Power	$x^6 = 1$	64	729	4096	15625	46656	117649	262144	531441
Second Surſolid, or 7th Power	$x^7 = 1$	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrate Squared, or 8th Po.	$x^8 = 1$	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cube-cubed, or 9th Power	$x^9 = 1$	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Surſolid Squared, or 10th Po.	$x^{10} = 1$	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
Third Surſolid, or 11th Power	$x^{11} = 1$	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
Square-Cube ſquared or 12th Po.	$x^{12} = 1$	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481

*Here follows the Method of Inveſtigation, or extracting the Roots of all Powers.**

THE Square Root having been ſpoken of before, I ſhall here begin with the Root of the Cube or third Power. And you are to take Notice, *Novitius, that in all the following Operations wherein* **e** *is above the ſecond Power,* that *Part muſt be rejected.*†

1. *Of the* CUBE ROOT.

Let the given Number whoſe Root is to be extracted, be $= b$, and let $x + e = \sqrt[3]{b}$: Then, if you involve $x + e$ to the third Power, that is, Cube it, you will have

	1	$x^3 + 3x^2e + 3xe^2 + eee = b$
1 rejecting eee, & $\div 3x$	2	$\frac{x^2}{3} + xe + e^2 = \frac{b}{3x}$
2 $- \frac{xx}{12}$	3	$\frac{x^2}{4} + xe + e^2 = \frac{b}{3x} - \frac{xx}{12}$
3 μυ 2	4	$\frac{x}{2} + e = \sqrt{\frac{b}{3x} - \frac{xx}{12}}$
4 $+ \frac{x}{2}$	5	$x + e = \frac{x}{2} + \sqrt{\frac{b}{3x} - \frac{xx}{12}} = \sqrt[3]{b}.$

---

* The following Theorems are the ſame in Effect, though not ſo eaſy in Practice, as ſome that were publiſhed by Dr. *Halley*, in the Philoſophical Tranſactions, which ſhall be treated of in the Supplement. Their Uſe is to find the Roots of pure Powers, by Approximation. I adviſe the Learner to omit them entirely, till he has acquired a Facility in the Management of Quadratic Equations. EDIT.

† Theſe Terms are rejected on account of their ſmall Value in Compariſon of the reſt. But it is to be noted, that the Root found this Way will not be exact, but only near the Truth. EDIT.

Now this Method will always hold good in every Operation, whether you ſuppoſe x + or — than it really is, as plainly appears from the next Work. For,

Let $x - e = \sqrt[3]{b}$

1 rejecting eee, & $\div 3x$	1	$x^3 - 3x^2e + 3xe^2 - e^3 = b$
	2	$\frac{x^2}{3} - xe + e^2 = \frac{b}{3x}$
$2 - \frac{xx}{12}$	3	$\frac{x^2}{4} - xe + e^2 = \frac{b}{3x} - \frac{xx}{12}$
3 μv 2	4	$\frac{x}{2} - e = \sqrt{\frac{b}{3x} - \frac{xx}{12}}$
$4 + \frac{x}{2}$	5	$x - e = \frac{x}{2} + \sqrt{\frac{b}{3x} - \frac{xx}{12}} = \sqrt[3]{b}$, as before.

2. *Of the Root of the* BIQUADRATE *or 4th* POWER.

Let	1	$x + e = \sqrt[4]{b}$
1 ⊚ 4	2	$x^4 + 4x^3e + 6x^2e^2 = b$
$2 \div 6x^2$	3	$\frac{x^2}{6} + \frac{4xe}{6} + e^2 = \frac{b}{6x^2}$
$2 - \frac{xx}{18}$	4	$\frac{xx}{9} + \frac{2xe}{3} + ee = \frac{b}{6x^2} - \frac{xx}{18}$
4 ωιn 2	5	$\frac{x}{3} + e = \sqrt{\frac{b}{6x^2} - \frac{xx}{18}}$
$5 + \frac{2x}{3}$	6	$x + e = \frac{2x}{3} + \sqrt{\frac{b}{6x^2} - \frac{xx}{18}} = \sqrt[4]{b}$.

3. *Of the Root of the* SURSOLID *or* 5*th* POWER.

Let	1	$x + e = \sqrt[5]{b}$
1 @ 5	2	$x^5 + 5x^4 e + 10x^3 e^2 = b$
$2 \div 10x^3$	3	$\frac{xx}{10} + \frac{xe}{2} + ee = \frac{b}{10x^3}$
$3 - \frac{3}{80}xx$	4	$\frac{xx}{16} + \frac{xe}{2} + ee = \frac{b}{10x^3} - \frac{3}{80}xx$
4 ω 2	5	$\frac{x}{4} + e = \sqrt{\frac{b}{10x^3} - \frac{3xx}{80}}$
$5 + \frac{5x}{4}$	6	$x + e = \frac{3}{4}x + \sqrt{\frac{b}{10x^3} - \frac{3xx}{80}} = \sqrt[5]{b}$

4. *Of the Root of the* CUBE *ſquared or* 6*th* POWER.

Let	1	$x + e = \sqrt[6]{b}$
1 @ 6	2	$x^6 + 6x^5 e + 15x^4 e^2 = b$
$2 \div 15x^4$	3	$\frac{xx}{15} + \frac{2xe}{5} + ee = \frac{b}{15x^4}$
$3 - \frac{2}{75}xx$	4	$\frac{xx}{25} + \frac{2xe}{5} + ee = \frac{b}{15x^4} - \frac{2xx}{75}$
4 ω 2	5	$\frac{x}{5} + e = \sqrt{\frac{b}{15x^4} - \frac{2xx}{75}}$
$5 + \frac{4x}{5}$	6	$x + e = \frac{4x}{5} + \sqrt{\frac{b}{15x^5} - \frac{2xx}{75}} = \sqrt[6]{b}$,
		&c.

And thus may you proceed to the Root of the 7*th*, 8*th*, 9*th*, 10*th*, *&c.* Powers.

Now from a due Conſideration, *Novitius*, of the above Work, (for I cannot expect you to be perfect in it yet) you may, by comparing the Roots thus inveſtigated, form Methods for finding of general Theorems to extract the Root of any higher Power without any troubleſome Operation.

Nov. I ſhall like to know that.

Phi. Obſerve then, firſt let us compare the four laſt Operations, and you will find the Fractions before x increaſe uniformly, and that the Numerator and Denominator of each is always Unity more added to each. Thus,

$\frac{1}{2}x\ \frac{2}{3}x\ \frac{3}{4}x\ \frac{4}{5}x\ \frac{5}{6}x$, *&c. &c.* as follows:

$$\frac{1}{2}x + \sqrt{\frac{b}{3x} - \frac{1}{12}xx} = \sqrt[3]{b}$$

$$\frac{2}{3}x + \sqrt{\frac{b}{6xx} - \frac{1}{18}xx} = \sqrt[4]{b}$$

$$\frac{3}{4}x + \sqrt{\frac{b}{10x^3} - \frac{3}{80}xx} = \sqrt[5]{b}, \text{ &c.}$$

So that they follow you ſee in a Series, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, *&c.* From hence follows,

$$\frac{1}{2}x + \sqrt{\frac{b}{\quad} - xx}$$

$$\frac{2}{3}x + \sqrt{\frac{b}{\quad} - xx}$$

$$\frac{3}{4}x + \sqrt{\frac{b}{\quad} - xx}$$

$$\frac{4}{5}x + \sqrt{\frac{b}{\quad} - xx}$$

$$\frac{5}{6}x + \sqrt{\frac{b}{\quad} - xx}, \text{ &c. &c.}$$

OBSERV. 2.

Again, If you compare the Power by which b is divided, you will find it increaſed by the continual Multiplication of x; and that the Co-efficient of the ſaid Power *(b)* is increaſed alſo by the continual Addition of 3, 4, 5, 6, 7, 8, &c. Hence therefore evidently ariſes

$$\frac{1}{2}x + \sqrt{\frac{b}{3x} - xx}$$

$$\frac{2}{3}x + \sqrt{\frac{b}{6x^2} - xx}$$

$$\frac{3}{4}x + \sqrt{\frac{b}{10x^3} - xx}$$

$$\frac{4}{5}x + \sqrt{\frac{b}{15x^4} - xx}$$

$$\frac{5}{6}x + \sqrt{\frac{b}{21x^5} - xx}, \text{ \&c. \&c.}$$

OBSERV. 3.

From what has been obſerved, it may eaſily be evinced, that the Fraction into which xx is multiplied, is found and produced by multiplying half the Fraction annexed to x into the whole Fraction annexed to b. Whence follows,

$$\frac{x}{2} + \sqrt{\frac{b}{3x} - \frac{1}{12}xx}$$

$$\frac{2x}{3} + \sqrt{\frac{b}{6x^2} - \frac{1}{18}xx}$$

$$\frac{3x}{4} + \sqrt{\frac{b}{10x^3} - \frac{3}{80}xx}$$

$$\frac{4x}{5} + \sqrt{\frac{b}{15x^4} - \frac{2}{75}xx}$$

$$\frac{5x}{6} + \sqrt{\frac{b}{21x^5} - \frac{5}{252}xx}$$

$$\frac{6x}{7} + \sqrt{\frac{b}{28x^6} - \frac{3}{196}xx}$$

Now in order to diſcover a Theorem, by which the Root of any Power may be extracted, you are to obſerve as follows.

NOTE 1.

That the *Denominator* of the Fraction into which x is multiplied, is always leſs by Unity (or 1) than the Index of the given Power, and alſo that the *Numerator* of the ſaid Fraction is leſs by one than the *Denominator*.

NOTE 2.

That the Index of the Power of x, by which b is divided, is always equal to the *Numerator* of the aforeſaid Fraction.

NOTE 3.

That the Co-efficient of the ſaid Power of x, is produced by multiplying $\frac{1}{2}$ the Index of the given Power into the ſaid Index leſs by 1.

LASTLY.

The Fraction into which xx is multiplied, is the Product of $\frac{1}{2}$ the Fraction annexed to x, and the whole numerical Fraction annexed to b.

EXAMPLE.

Let the Thorem of the $\sqrt[30]{b}$ be required; then by Note 1. $\begin{matrix}29 - 1 = \\ 30 - 1 =\end{matrix}\frac{28}{29}$ Fraction of x.

And by Note 2. follows $\frac{b}{x^{28}}$

By the third Note, $15 \times \overline{30 - 1} = 435$, therefore $\frac{b}{435x^{28}}$. And

By the laſt, $\frac{14}{29} \times \frac{1}{435} = \frac{14}{12615}$

The Theorem then when compleated is $\frac{28}{29}x + \sqrt{\frac{b}{435x^{28}} - \frac{14}{12615}xx} = \sqrt[30]{b}$. Again,

Let the Theorem of $\sqrt[12]{b}$ be required.

Firſt, $\frac{11-1}{12-1} = \frac{10}{11}x$

2dly, $\frac{b}{x^{10}}$

3dly, $6 \times \overline{12-1} = 66$, which compleated is $\frac{10}{11}x + \sqrt{\frac{b}{66x^{10}} - \frac{5}{726}xx} = \sqrt[12]{b}$

From a little Obſervation and Practice, *Novitius*, you may, from theſe Examples, improve yourſelf further in theſe Things.

Nov. I could do it the better if you would pleaſe to work me a few Problems.

Phi. I will, and ſet a few for Trial.

Here follows ten PROBLEMS, *ſhewing the Manner of compleating the Square and extracting the Root thereof.*

PROBLEM I.

There are three Numbers in ∺; the Mean is 9, and the Difference of the Extremes is 24; I demand the Numbers?

1 | x for the leſs Extreme
2 | $x + 24$ the greater
3 | $xx + 24x$ their Product (Q)
4 | $xx + 24x = 81$, the □ of the Means.

Now x being found with the Co-efficient 24 before it, you muſt firſt compleat the Square before you can tell the Value of x, thus: take $\frac{1}{2}$ the Co-efficient, *viz.* 12, whoſe Square is 144, put it on both Sides of the Equation, then it will be *

---

* *N. B.* I give this as one general Rule for all the other Examples.

5 $xx+24x+144=81+144$, *i.e.* $=225$. Then extract the Square Root on both Sides of the Equation, and you have
6 $x+12=15$. (*See Example* 1. *pa.* 191.) Now take 12 from each Side of this new Equation, and you have
7 $x=3$, the leſs Number.
8 Laſtly, $x+24=3+24=27$, the greater N°.

PROOF.

$3 : 9 :: 9 : 27$; for $3\times27=9\times9$.

PROBLEM II.

A certain Number of young Rakes agreed to have an Evening's Frolick at a Country Fair, and went into an Inn or Tavern and eat and drank freely; but upon calling for the Reckoning (which amounted juſt to 7 Pounds and 10 Shillings) 4 of them ſneaked away, by which Means thoſe left were obliged to pay 10s. a Piece more; I demand the Number at firſt, and the Number left.

1 x N° at firſt
2 $\frac{150}{x}$ the Share of each
3 $x-4$ thoſe left
4 $\frac{150}{x-4}$ Share of thoſe left
5 $\frac{150}{x-4}-10=\frac{150}{x}$ (Q.) Then $\times\,\overline{x-4}$,
6 $150-10x+40=\frac{150x-600}{x}$
7 $150x-10xx+40x=150x-600$. Or
8 $10xx-40x=600$. That is, ÷ 10,
9 $xx+4x=60$. Now C $\square$, *viz.* compleating the Square,
10 $xx-4x+4=60+4=64$
11 $x-2=\sqrt{64}=8$
12 $x=8+2=10$, the Number at firſt
13 $x-4=10-4=6$, thoſe left.

PROOF.

PROOF.

10 Perſons, at 15*s.* each = 150*s.* = £7. 10.
6 Perſons, at 25*s.* each = 150*s.* = £7. 10.

PROBLEM III.

What Number is that from whoſe Double 10 being ſubtracted, the Square of the Remainder leſs 1 will be equal to 7 Times the ſaid Number?

1 x for the Number
2 $2x - 10$ whoſe □ is
3 $4xx - 40x + 100$. Then (Q.)
4 $4xx - 40x + 100 - 1$ (or 99) $= 7x$. That is
5 $4xx - 47x = -99$. Then $\div 4$
6 $xx - 11.75x = -24.75$. C □ in the ſame Manner as before, by adding the Square of $\frac{1}{2}$ the Coefficient of x to both Sides of the Equation, and you will have,
7 $xx - 11.75x + \overline{5.875}^2 = -24.75 + \overline{5.875}^2$, *i. e.* $xx - 11.75x + 34.515625 = 9.765625$, (for $-24.75 + 34.515625 = 9.765625$;) and extracting the Square Root on both Sides,
8 $x - 5.875 = \sqrt{9.765625}$, *viz.* 3.125. And laſtly,
9 $x = 5.875 + 3.125 = 9$, the Number.

Note, In this Solution, the Manner of compleating the Square, and extracting the Root, as they ſtood in the former Editions of this Book, was unintelligible.

EDIT.

PROOF.

$9 \times 2 = 18 - 10 = 8$. Now $8 \times 8 = 64$; and $64 - 1$ is $= 63 = 7 \times 9$.

PRO-

PROBLEM IV.

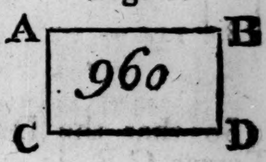
A, Executor to *B*, finds among other Things a Promiſſory joint Note of £. 174 due from *C* and *D*, and makes a Demand of it; but *C* and *D* not being able directly to pay it, 'twas agreed upon to take it as follows: *C* was to pay £. 8 every Day; and *D* was to pay £. 1 the 1ſt Day, 2 the 2d, 3 the 3d, &c. I demand in how long Time the Debt will be paid?

1 | Put x for the Days, or Time; then
2 | $8x =$ the Number of Pounds paid by *C*.
3 | $\frac{xx+x}{2} =$ to what *D* pays. And their Sum
4 | $\frac{xx+x}{2} + 8x = 174$, (Q.)
5 | $xx + x + 16x = 348$. C. □
6 | $xx + 17x + 72.25 = 420.25$. Extr. Roots
7 | $x + 8.5 = 20.5$, or $x = 20.5 - 8.5 =$
8 | 12 Days. So that *C* paid 96 and *D* 78 £.

PROBLEM V.

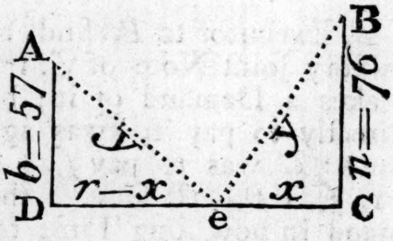
There is a Field in the Form of a regular Parallelogram ABCD; the Length exceeds the Breadth by 16 Rods or Perches, and it contains 960 Square Rods; I demand the Length and Breadth of the Field?

Fig. 1.



1 | x for the Breadth
2 | $x + 16$ Length
3 | $xx + 16x$ Area (Q.)
4 | $xx + 16x = 960$. C □
5 | $xx + 16x + \underline{64} = 960 + 64 = 1024$
6 | $x + 8 = \sqrt{1024} = 32$; and $x = 32 - 8 = 24$, the Breadth
7 | $x + 16 = 40$, the Length

PROBLEM VI. Fig. 2.



A certain Gentleman had in his Garden a Fountain, in the Middle of which ſtands a Lion; at unequal Diſtances from the Centre of the Fountain he built two Towers A and B, whoſe Baſes DC are in an Horizontal Plane, or even with the Head of the Lion at *e:* The Height of the Tower AD is 57 Feet, and BC 76, and the Diſtance between them (*viz.* DC) is 114 Feet: Now the Head of the Lion (at *e*) is equi-diſtant from the Top of each Tower: I demand the Diſtance from the Lion to the Baſe of each of the Towers?

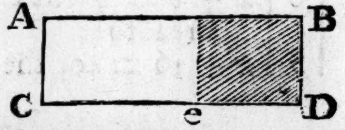
1 | x for the leaſt Diſtance eC, and let $r-x=De$
2 | $nn+xx=yy$ per Figure, and 47 I. Euclid.
3 | $bb+rr-2rx+xx=yy$ per Figure
4 | $nn=bb+rr-2rx$
5 | $nn+2rx=bb+rr$
6 | $2rx=bb+rr-nn$
7 | $x=\dfrac{bb+rr-nn}{2r}=45\frac{11}{12}$, And
8 | $114-x=68\frac{1}{12}$

Sum 114

So that the Diſtance from the Lion at *e*, to the Tower CB, is $45\frac{11}{12}$ Feet, and from the Tower DA $68\frac{1}{12}$ Feet.

Here follows a few more Queſtions to exerciſe the Learner.

PROBLEM VII. Fig. 3.



There is a Piece of Ground, ABCD, in the Form of a Parallelogram; the Length is three Times the Breadth: This Piece of Ground is deſigned for two Gardens; the

Length of the Kitchen-Garden C *e* is 18 Rods, and the Content of the Flower-Garden D B *e* is 120; I demand the Length and Breadth of the whole Ground.

PROBLEM VIII.

There are two Lines A and B in ſuch Proportion that the Double of the firſt (*viz.* A) with the Triple of the ſecond (B) will make 60; and double the Square of the ſecond with Triple the Square of the firſt, will make 14 Times as much; I demand the Length of the Lines A and B.

PROBLEM IX.

There is a certain Number conſiſting of 2 Figures, and it is equal to 4 Times the Sum of its Digits (*viz.* the Sum of the 2 Figures) and if you add 18 to the Number, the Digits will be inverted, *viz.* the firſt Figure will ſtand laſt, and the laſt firſt; I demand the Number?

PROBLEM X.

I happen'd one Ev'ning with a Tinker to ſit,
(Whoſe Tongue ran a great deal too faſt for his Wit)
He talk'd of his Art with Abundance of Mettle;
Then I aſk'd him to make me a flat-bottom'd Kettle,
The Top and the Bottom Diameter to be,
In ſuch juſt Proportion as five *is to* three:
Twelve Inches the Depth (I propos'd) and no more,
And to hold in Ale Gallons Sev'n leſs than a Score.
He promis'd to do it, and forthwith to work went;
But when he had done it, he found 'twas too ſcant:
He alter'd it then; but too big he has made it;
For tho' it holds right, the Diameters fail it:
Thus making it often too big and too little,
The poor Tinker at laſt had quite ſpoil'd his Kettle;
But he ſwore that he'd bring his ſaid Purpoſe to paſs,
Or elſe he would ſpoil ev'ry Ounce of his Braſs:
Now to keep him from Ruin, I pray find him out
The Diameter's Length?—For he'll ne'er do't I doubt!

Note 1. This Queſtion was ſent to the AUTHOR about two Years ago; and you have the ſame Numbers with very little Alteration of the Words.

Note 2. By the EDITOR. This Queſtion was taken from the Ladies Diary for the Year 1711.

Thus, *Novitius*, I hope you will be kind to young *Tyro*, and inſtruct him in all that lies in your Power, as I have been wanting in nothing that might be of Service to you.

Nov. Sir, You may depend upon it; and permit me to aſk you but one Thing more, and I will trouble you no longer.

Phi. What is that, pray?

Nov. Only to give me the Work of the different Content of a Piece of Timber or Stone in the Form of a Cylinder, and Parallelopiped, for ſome ſay there is no Difference at all.

Phi. 'Tis a common Miſtake, but there is very near $5\frac{1}{2}$ Feet Difference, *Novitius*, the Content of the regular Parallelopiped being juſt 20 Feet, and the Cylinder 25 Feet $\frac{4}{10}$. But pray, *Novitius*, let me know the Reaſon of your aſking this Queſtion, for your ſeem to be very earneſt about it?

Nov. To tell you the Truth then, Sir, there was ſome ſmall Diſpute between two or three of us concerning it; but I could not make them ſenſible there is any Difference at all.

Phi. But why did you not work them both by Figures, and that would have convinced them?

Nov. I did, and make it the ſame as you do; but they would not be ſatisfied with *that*, which occaſioned a ſmall Bet between us to be left to your Determination.

Phi. If you did it the right Way ſurely they could not be ſo ignorant! Let me ſee your Method of doing it?

Nov. Firſt for the ſquare Tree, that being 48 Inches Circumference, conſequently has 12 Inches upon every Side. Now 12 multiplied by 12 makes 144, the ſuperficial Content; and this multiplied by 20 the Length, and divided by 144, gives juſt 20 Feet the Content.

Phi. Very right; and how did you proceed with the Cylinder?

Nov. A Cylinder having a Circle for its Baſe (and being 48 Inches Circumference) I find firſt the Diameter, thus: As 3.1416 to 1, ſo is the Circumference (48) to the Diameter (15.2788 Inches). Then to find the ſuperficial Content at the End, I multiply Half the Diameter, (*viz.* 7.6394) by Half the Circumference, (*viz.* 24) and it gives 183.3456 Inches the Area at the End. This divided by 144 gives 1.2732 Feet; and this multiplied by 20 the Length, gives 25.464 ſolid Feet, which is nearly 25½ as you obſerved before.

Phi. Very rightly performed; and would not this ſatisfy them, do you ſay?

Nov. No, indeed: they ſay all the calculated Tables in Timber-Meaſure prove the Contrary: So, as I obſerved before, it is left to you to decide.

Phi. To oblige you, *Novitius*, I will ſhew you (by and by) a Method that will not fail to convince them. But firſt I will tell you the Reaſon of this common Error; for you muſt note, you are not the only Perſon that has been Witneſs to this Folly. As to all the ſet Tables, they are calculated for ſquare-ſided Timber only, according to Cuſtom (for we are not to ſuppoſe every pretended Meaſurer a *Geometrician)* and in this the Pen and the Tables will agree; and the Reaſon is this, they girt the Tree round, then take the fourth Part of that Circumference (vulgarly called the Girt by ſome) and multiply it by itſelf, then by the Length of the Tree; after which they divide it by 144, and it gives the Content in ſolid Feet; but then, as I ſaid before, it is only for ſquare-ſided Timber that this Method holds good. For of all other ſhaped Timber the Content will be more or leſs as I ſhall demonſtrate hereafter, that will not fail, I believe, to convince your Friends of their Error.

If indeed the Buyer and Seller agree according to a cuſtomary Way of meaſuring any Thing, we have no Buſineſs to meddle; but when we are called upon to do Juſtice between both, we muſt then proceed according to the juſt Rules of Arithmetic; which ought not in any Reſpect to give Way to Things introduced merely by Prejudice or Ignorance, which may very well be called the Nurſes of idle Cuſtom, as you may ſee in a Series of Inſtances beſides the Caſe before us. But to give one only,

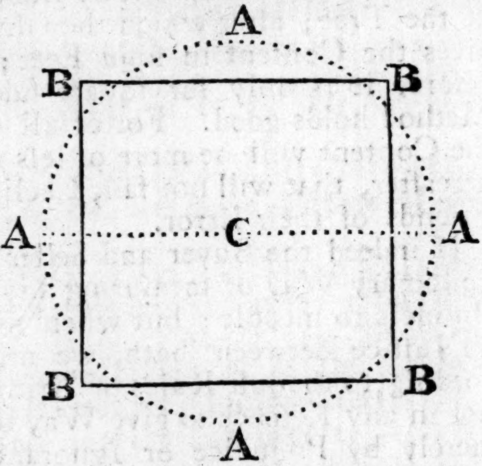
I have heard a great many pretended Meaſurers affirm, that take a round Piece of Timber, and let four

Slabs be ſawn off it, and even *then* it will contain more ſolid Feet than it did before. The *Engliſh* of which is, if I give you Two-pence out of a Shilling, I ſhall then have 14*d.* in Hand.——What Stupidity is here! Again, in a ſquare Tree 48 Inches round, it is plain one Side is but 12 Inches; but in a round Tree that is 48 Inches Circumference, the Side of a Square equal thereto will be ſomewhat above 13$\frac{1}{2}$ Inches,* but the inſcribed Square will be on each Side but about 10$\frac{8}{10}$ Inches.

However, I ſhall leave Arithmetic, and demonſtrate it by one plain geometrical Figure only, which I never knew fail to convince this Sort of People; becauſe they can ſee the Reaſon of it directly upon looking at the Scheme.

Demonſtration.

As it has been proved that a Circle 48 Inches Circumference, is 15,2788 Diameter, the Semi-Diameter muſt be 7.6394. From any Scale of equal Parts therefore take off with your Compaſſes 7.6 Inches, and from C the Center, deſcribe the Circle A, A, A, A, whoſe Circumference will then be 48. Then from the ſame Scale take off 12 and make this the Side of a Square; then complete the Square B, B, B, B, whoſe Perimeter will alſo be 48 Inches. Now ſuppoſing this Square to be laid upon the Circle, does it not evidently appear by the Figure itſelf, that the *Area* or ſuperficial Content of the Circle is larger than the Square: For though the Square hangs over the Circle at the Points B, B, B, B, yet the four Areas or Segments of the Circle A, A, A, A, are each of them larger than



---

* *Find the Area of the Circle, and extract the ſquare Root thereof gives the Side of the Square equal to the Circle.*

the former. Conſequently therefore the *Area* of a Circle is larger than the *Area* of a Square, whoſe Perimeter is equal to the Circumference of the Circle; and if the ſuperficial Content be greater, it is out of Diſpute that the ſolid Content is alſo greater.

Nov. This is a plain Demonſtration indeed!

Phi. To be ſure it is much the eaſieſt Way: For ſuch as are ignorant of the Square and Cube Root, only think you are impoſing upon upon them when you work ſuch Queſtions at large; but here they are convinced directly.

Nov. They are ſo.

Phi. From hence then it is evident, That a Circle is larger in Area than any other Figure having the ſame Circumference. And all Polygons are nearer the Area of a Circle of equal Ambit, according to the Number of Sides; (as a Triangle, Square Pentagon, Hexagon, *&c. &c. &c.*) for the more the Sides the nearer the Circle, but they never can be quite ſo, for this Reaſon, becauſe a Curve Line is longer than a ſtraight one.* Again,

You are to obſerve, that the Side of the inſcribed Square (in the aforeſaid Circle) will be nearly 10.8 Inches, and its Content 16.21 Feet; and the Content of the circumſcribed Square will be juſt the double, *viz.* 32.42 Feet. The Content of the Triangle (48 Inches round) 15.4 Feet. That of the Square juſt 20; that of the Hexagon 23 Feet.

Nov. Dear *Philomathes*, I heartily thank you.

Phi. You are heartily welcome, only do you communicate to *Tyrunculus* what I have ſhewn you; for if I ſee you both diligent, I intend (as ſoon as I have done with young *Tyro* in common *Arithmetic*) to inſtruct you in the Rudiments of *Geography* and the Uſe of the *Globes*. In the mean Time, *Novitius*, I bid you a hearty Farewel.

Nov. Sir, I am your obliged humble Servant.

---

* It is well known to Mathematicians, that the Area of a Circle is greater than the Area of any other Figure of equal Ambit; but the Argument for it above given is not concluſive. EDIT.

[illegible] Consequently therefore the [illegible] [illegible]

This is a [illegible]

[illegible] is the [illegible] of the Square [illegible] that [illegible] their [illegible]

[illegible] in Acceleration [illegible] And [illegible] Globe [illegible] as a Particle [illegible]

In order [illegible] of the [illegible]

You [illegible] only on [illegible] com- [illegible] you [illegible] with many [illegible]

[illegible]

[illegible]

A
SUPPLEMENT
TO THE
Young Algebraiſt's Companion:

CONTAINING,

I. The Methods of exterminating two, three, or more unknown Quantities out of an Equation.

II. Some further Obſervations on Quadratics: In which it is proved that every ſuch Equation has two Roots.

III. A Definition of Surds, according to Dr. *Saunderſon*; and ſome uſeful Rules relating to them.

IV. Some further Obſervations on the Extraction of Roots: in which the Uſe of Logarithms is recommended and illuſtrated.

Now in the Preſs,

MATHEMATICAL ESSAYS,

On Several Subjects:

CONTAINING

NEW IMPROVEMENTS AND DISCOVERIES

IN THE

MATHEMATICS.

By the Rev. JOHN HELLINS.

N. B. The Work is in Quarto, and the Letter and Paper are exceedingly good.—The Price, to Subſcribers, is Five Shillings; to Non-Subſcribers it will be more.

February 21, 1787.

PREFACE.

IN the foregoing Part of this Book, the Author, Mr. FENNING, *has treated only of Equations that have one unknown Quantity; which, indeed, was ſufficient for the Solutions of the Problems he has given. But there are Problems that require the Uſe of two, three, or more unknown Quantities; and even ſome of thoſe Problems that are ſolved by Means of one unknown Quantity, may be yet eaſier done by Means of two or more. It was therefore thought, by ſeveral Teachers of the Mathematics, that the Management of Equations that contain two, three, or more unknown Quantities, and the Solutions of ſome Problems that produce ſuch Equations, would be a valuable Addition to the* YOUNG ALGEBRAIST'S COMPANION. *In Compliance with their Wiſhes I have written this Supplement, which is divided into four Sections.*

Sect. I. *Contains the Methods of exterminating two, three, or more unknown Quantities out of an Equation, together with the Solution of ſome Problems by Way of Illuſtration.*

Sect. II. *Contains ſome further Obſervations on Quadratics; in which it is proved, that every ſuch Equation hath two Roots.*

Sect. III. *Treats of Surds; their Uſe in Calculations, &c.*

Sect. IV. *Contains ſome further Obſervations on the Extraction of Roots; in which the Uſe of Logarithms is recommended and illuſtrated; and the Advantage of computing by thoſe artificial Numbers, even to Algebraiſts, is clearly pointed out.*

The Method of Dialogue is laid aſide, as it may reaſonably be ſuppoſed, that the Learner cannot now want ſo many Words in the Explanation of any Matter as at his firſt Entrance on theſe Studies. However, I have endeavoured to make every Thing I have treated of eaſy to be underſtood.

GREEN'S NORTON,
in the County of Northampton,
December 1, 1786.

J. HELLINS.

SUPPLEMENT.

SECT. I.

Of exterminating two, three, or more unknown Quantities out of an Equation.

"WHEN two or more Equations are propoſed, involving as many unknown Quantities, theſe "Equations muſt firſt be prepared by freeing them from "Fractions whenever there are any, and by ordering "every particular Equation ſo, that all the unknown "Quantities may poſſeſs one Side of the Equation, and "ſuch as are known, the other; or elſe, that all the "Quantities may poſſeſs one Side of the Equation, and "a Cypher the other. It will alſo be convenient, that "in every particular Equation, the unknown Quantities be placed in the ſame Order."

The above Quotation is from Dr. *Saunderſon's Algebra*, vol. 1, page 136. I now proceed to deliver the Rules of Extermination.

When there are two unknown Quantities, x, *and* y, *and two Equations*,

RULE I.

Find a Value of one of them in each of the Equations, and put theſe two Values equal to each other, and you will have a new Equation which ſhall contain but one unknown Quantity. Or,

RULE II.

Find a Value of one of theſe unknown Quantities from the firſt Equation, and ſubſtitute that Value for it in the ſecond, and there will ariſe a new Equation, in which there will be but one unknown Quantity. Or,

RULE III.

Either multiply or divide the two given Equations by any two Numbers or Quantities that will make the Co-

efficient

efficient of the Quantity you would exterminate the ſame in both Equations, and then, if the Signs be like, ſubtract *the one Equation from the other,—if they be unlike,* add *the Equations together, and you will have a new one, in which there will be but one unknown Quantity.*

EXAMPLE I.

Given $x+y=4$
And $x-y=2$ } Quere x and y?

Solution by RULE I.

From the firſt Equation we have, by tranſpoſing y, $x=4-y$; and from the ſecond Equation, $x=2+y$.

Now put theſe two Values of x equal to each other, and there will be a new Equation, from which x will be exterminated, thus:

$$2+y=4-y.$$

And then, by tranſpoſing 2 and $-y$, we get

$$2y=4-2=2,$$

And $y=\frac{2}{2}=1.$

The Value of y being now known, that of x will be eaſily obtained from either of the Equations in which it is expreſſed in Terms of y and known Quantities. For ſince we have $x=4-y$, and y is known to be $=1$, we have

$$x=4-1=3.$$

Or, if we take the other of the Equations that expreſſes the Value of x in Terms of y and known Quantities, we ſhall have $x=2+y$, $=2+1$, $=3$, as before.

The unknown Quantity y might have been exterminated in like Manner, which I will now do by Rule II.

1 2 Given $\left\{\begin{array}{l} x+y=4 \\ x-y=2 \end{array}\right\}$ to find x and y.

By $\varphi\ x$ in the 1ſt Equation, we get
3 $y=4-x$. Then ſubſtitute this Value of y for it in the 2d Equation, and we have,
4 $x-4+x=2$, *i. e.* $2x-4=2$. And laſtly, by $\varphi\ 4$, and dividing by 2, we have,
5 $x=\frac{6}{2}=3$, as before.

The ſame done by RULE III.

Add the 2d Equation to the firſt, and the Sum will be

$$2x+y-y=4+2,\ i.\ e.$$
$$2x=6,\ \text{and}\ x=6\div 2=3.$$

Again, ſubtract the ſecond Equation from the firſt, and the Remainder will be $2y=2$, and conſequently $y=1$.

And this is evidently the eaſieſt Way of finding the Values of the unknown Quantities in the above Equations. It is to be noted, however, that ſometimes one of theſe Rules will have the Advantage, and ſometimes the other.

EXAMPLE 2.

1 2 Given $\left\{\begin{array}{l} 4x+3y=29 \\ 2x+7y=31 \end{array}\right\}$ to find x and y.

By $\varphi\ 3y$ in the firſt Equation we get
3 $4x=29-3y$, and $x=\frac{29-3y}{4}$. Again, by $\varphi\ 7y$ in the ſecond Equation, we get
4 $2x=31-7y$, and $x=\frac{31-7y}{2}$. Now put theſe two Values of x equal to each other, and we ſhall have
5 $\frac{29-3y}{4}=\frac{31-7y}{2}$. And then, by multiplying both Sides of the Equation by 4, we ſhall have

6 $29 - 3y = 62 - 14y$; for any Quantity multiplied by 4 and divided by 2 is = the ſame Quantity multiplied by 2. Now, by Tranſpoſition, we have

7 $14y - 3y = 62 - 29$; *i. e.* $11y = 33$, and

8 $y = 33 \div 11 = 3$. The Value of *y* being now found, the Value of *x* may be eaſily diſcovered from either the 3d or 4th Steps. From the 3d Step we have

9 $x = \dfrac{29 - 3y}{4} = \dfrac{29 - 9}{4} = \dfrac{20}{4} = 5$.

But the Values of *x* and *y* in the above Equations may be quicker diſcovered by Rule III.

Since the Co-efficient of *x* in the firſt Equation is double the Co-efficient of it in the ſecond, multiply the ſecond Equation by 2, and it becomes $4x + 14y = 62$; then take the firſt Equation from it, $4x + 3x = 29$,

There remains * $11y = 33$,
and conſequently $y = 3$.

Example 3.

1 Given $x + y = 20$
2 and $x : y :: 2 : 3$ } Quere *x* and *y*?

By multiplying Means and Extremes in the ſecond Equation, we get

3 $3x = 2y$, and $x = \dfrac{2y}{3}$. And by tranſpoſing *y* in the firſt Equation, we have

4 $x = 20 - y$. Put theſe two Values of *x* equal to each other, we have

5 $\dfrac{2y}{3} = 20 - y$, from which we get

6 $2y = 60 - 3y$. And φ $3y$,

7 $5y = 60$, and $y = 60 \div 5 = 12$. Laſtly,

8 $x = 20 - y = 20 - 12 = 8$.

Theſe Examples may be ſufficient to illuſtrate the Methods of exterminating two unknown Quantities out of an Equation.

When there are three unknown Quantities, x, y, *and* z, *in an Equation, the ſame Rules are to be uſed that were given in Page* 219, *for the Extermination of two unknown Quantities; only it is to be obſerved, that there muſt be given three Equations, or the Values of thoſe Quantities will not, in general, be determined.*

EXAMPLE.

1, 2, 3

$$\text{Given}\left\{\begin{array}{l} x + y + z = 12 \\ x + 2y + 3z = 20 \\ \frac{x}{3} + \frac{y}{2} + z = 6 \end{array}\right\}\text{to find } x, y, \text{ and } z.$$

By Reduction and Tranſpoſition, we get theſe three Equations,

4 $x = 12 - y - z$

5 $x = 20 - 2y - 3z$

6 $x = 18 - \frac{3y}{2} - 3z$. Now ſince theſe three Equations are equal among themſelves, we have, by taking 5th and 6th Steps,

7 $18 - \frac{3y}{2} - 3z = 20 - 2y - 3z$, or

$18 - \frac{3y}{2} = 20 - 2y$, from which, by clearing the Equation of Fractions, and Tranſpoſition, we get,

8 $4y - 3y = 40 - 36$, and

9 $y = 4$. Having obtained the Value of y, let us write that Value for it in the 4th and 5th Equations, and we ſhall have

10 $x = 12 - 4 - z$, *i. e.* $x = 8 - z$, and

11 $x = 20 - 8 - 3z$, *i. e.* $x = 12 - 3z$. Now put theſe two Values of x equal to each other, and there will be

12 $8 - z = 12 - 3z$; from which, by Tranſpoſition, we get,

13 $2z = 4$, and $z = 2$.
The Values of y and z being now obtained, it remains only to find the Value of x, which may be done by writing for y and z their Values in either the 4th, 5th, or 6th Equation; but eaſier ſtill by writing for z its Value in the 10th Step, *viz.* $x = 8 - z$, which becomes
14 $x = 8 - 2 = 6$.

Note, The diligent Learner may perceive, from what has been done in ſome former Examples, that the Values of theſe unknown Quantities might have been found in a Manner ſomewhat different and eaſier. One might have taken the 6th Equation from the 5th, and the Remainder would have been $0 = 2 - \frac{y}{2}$, where it immediately appears that $y = 4$.

Again, if the 10th Step be taken from the 11th, the Remainder will be $0 = 4 - 2z$, from which $2z$ appears to be $= 4$, and $z = 2$.

With reſpect to the Extermination of four or more unknown Quantities, it is performed in a Manner altogether ſimilar to that above; ſo that any Example of it here would be unneceſſary. It may, however, be proper to note, that as many unknown Quantities as are uſed in the Solution of any Queſtion, ſo many Equations ought there to ariſe from the Data; otherwiſe, in general, it will not be limited.

I will now apply the above Rules to the Solution of ſome Problems. And here, I conceive, it will be moſt uſeful and agreeable to the Learner, to ſee ſome of thoſe Problems that were given in the former Part of this Book ſolved by Means of two or more unknown Quantities.

PROBLEM V.

Solved by Means of Equations that have two unknown Quantities. See Page 138.

Put x for the Quarters of Wheat, and y for thoſe of Barley; Then, (Q.) $x + y = 60$ Quarters. And ſince the Wages for threſhing the Wheat, per Quarter, was 12d. or a Shilling, the Money to be received for x Quarters will be $= x$ Shillings; and the Money to be received for y Quarters of Barley, at 6d. per Quarter, will be $= \frac{1}{2} y$ Shillings; and the Sum of theſe muſt be = the Wages received; *i. e.* $x + \frac{y}{2} = 48$. This Equation × by 2 gives $2x + y = 96$; and if the firſt be taken from it, we ſhall have $x = 96 - 60 = 36$, the Quarters of Wheat; and then, $y = 60 - 36 = 24$, the Quarters of Barley.

PROBLEM VI.

Solved by Means of two unknown Quantities.
See Pa. 139.

Put x for the Days he worked, and y for thoſe he was idle; then,
1 $x + y = 40$, (Q.) and
2 $20x =$ Pence he earned
3 $8y =$ Pence forfeited. The Difference of theſe is = the Pence received, which were 184; *i. e.*
4 $20x - 8y = 184$. Now multiply the firſt Equation by 8, and we have
5 $8x + 8y = 320$; and if this Equation be added to the laſt we ſhall have
6 $28x = 504$, and $x = 504 \div 28 = 18$, the Days he worked; and conſequently
7 $y = 40 - 28 = 22$, the Days he was idle.

PROBLEM VII.

Solved by Means, &c. See Page 140.

Put x for the Number of A's Crowns, and y for B's; then, if B gives A 3, their Numbers will be $x + 3$, and $y - 3$, which (Q.) are equal; *i. e.*
1 $x + 3 = y - 3$, from which we get $y = x + 6$.
But if A had given 3 to B, their Numbers would have been $x - 3$, and $y + 3$, which (Q.) are as 1 and 5. Therefore we have

2 $5x - 15 = y + 3$; and ſince y is $= x + 6$, we have, by ſubſtituting this Value for y in the ſecond Equation,
3 $5x - 15 = x + 9$; from which, by Tranſpoſition, we get,
4 $4x = 24$, and $x = 24 \div 4 = 6$. And then
5 $y = x + 6 = 12$.

PROBLEM XXV.

Solved by, &c. See Page 159.

Put x for the Sum placed out at 5 *per Cent.* and y for that at 6 *per Cent.* then, (Q.)
1 $x + y = 135$. And 15 Years Intereſt of x, at 5 *per Cent.* will be $\frac{75x}{100} = \frac{3}{4}x$; and 15 Years Intereſt of y, at 6 *per Cent.* will be $\frac{90x}{100} = \frac{9}{10}y$; and their Sum,
2 $\frac{3}{4}x + \frac{9}{10}y = 105$ (Q.) This Equation out of Fractions is
3 $30x + 36y = 4200$. If now the firſt Equation multiplied by 30 be ſubtracted from the third, there will remain,
4 $6y = 4200 - 4050 = 150$, and
5 $y = 150 \div 6 = 25$£. Laſtly,
6 $x = 135 - 25 = 110$£.

PROBLEM XX.

Solved by Means of three unknown Quantities.
See Page 155.

Let the Debts of A, B, and C, be denoted by x, y, and z, reſpectively; then,

$$\begin{array}{rl}1 & x + y = 13\frac{1}{2}\\ 2 & x + z = 31\frac{1}{2}\\ 3 & y + z = 37\frac{1}{2}\end{array}\left.\vphantom{\begin{array}{l}a\\a\\a\end{array}}\right\}\text{ Per Queſtion.}$$

Now, if the firſt Equation be taken from the ſecond, there will remain,

4 $-y + z = 18$; and if this be added to the third Equation, the Sum will be

5 $2z = 55\frac{1}{2} = \frac{111}{2}$, and $z = \frac{111}{4} = 27\frac{3}{4} = £.\ 27 - 15$. We have now only to ſubtract z from the firſt Side of the ſecond and third Equations, and its Value, £. 27 - 15, from the ſecond Side, and we ſhall have

6 $x = £.\ 31 - 10 - £.\ 27 - 15 = £.\ 3 - 15$

7 And $y = £.\ 37 - 10 - £.\ 27 - 15 = £.\ 9 - 15$

PROBLEM XXX.

Solved by Means of four unknown Quantities.
See Page 163.

Let the Number of Pounds ſnatched by A, B, C, and D, be denoted by u, v, x, and y, reſpectively; then,

$$\left.\begin{array}{ll} 1 & u + v + x + y = 475 \\ 2 & u + 4 = v - 4 \\ 3 & v - 4 = 4x \\ 4 & 4x = \frac{y}{4} \end{array}\right\} \text{Per Queſtion.}$$

Now, it is evident, that, if we can find the Value of any one of theſe four unknown Quantities, the Values of all the reſt will be quickly known. It is alſo plain, from what has been done before, that we may retain which of theſe Letters we pleaſe.

To exterminate all but u: From the ſecond Equation we have

5 $v = u + 8$. And becauſe it appears by the ſecond and third Equations, that $u + 4 = 4x$, we have

6 $x = \frac{u}{4} + 1$. And ſince $u + 4 = \frac{y}{4}$,

7 $y = 4u + 16$. Now, write theſe Values in the firſt Equation, and we have

8 $u + u + 8 + \frac{u}{4} + 1 + 4u + 16 = 475$. Take away 25 ($= 8 + 1 + 16$) from both Sides of the Equation, and collect the like Terms together, and there will be

9 $6u + \frac{u}{4} = 450$; which being reduced, gives

10 $u = 1800 \div 25 = 72$. We have now the Values of the other Quantities, thus:

$$v = 72 + 8 = 80;$$

$$x = \frac{72}{4} + 1 = 19;$$

$$\text{And } y = 288 + 16 = 304.$$

I will now add a few new Queſtions and their Solutions.

QUESTION I.

What Fraction is that, to the Numerator of which if 1 be added, it ſhall be $= \frac{1}{9}$; but if 1 be added to the Denominator, it ſhall be $= \frac{1}{10}$?

Solution.

Let $\frac{x}{y}$ repreſent the Fraction; then,

$$\left.\begin{array}{ll} 1 & \dfrac{x+1}{y} = \dfrac{1}{9} \\ 2 & \dfrac{x}{y+1} = \dfrac{1}{10} \end{array}\right\} \text{Per Queſtion.}$$

Theſe Equations being cleared of Fractions give

3 $9x + 9 = y$, and

4 $10x = y + 1$. Subtract the third Equation from the fourth, and the Remainder will be

5 $x - 9 = 1$, or $x = 10$; and then, from the third Equation we have

6 $y = 9x + 9 = 99$; and conſequently, the Fraction required is $\frac{10}{99}$.

QUESTION II.

The Sum of two Numbers is 7, and the Difference of their Squares is 21; what are thoſe Numbers?

Solution.

Let x be the greater and y the leſs of the Numbers; then,

1 $x + y = 7$
2 $x^2 - y^2 = 21$ } Per Queſtion.

From the firſt Equation,

3 $x = 7 - y$; and if this Value be written for x in the ſecond Equation, we ſhall have

4 $49 - 14y + yy - yy = 21$; *i. e.* $49 - 14y = 21$, from which y is eaſily diſcovered to be

5 $= 28 \div 14 = 2$; and then

6 $x = 7 - 2 = 5$.

Another Solution.

Divide the ſecond Equation by the firſt, and the new Equation will be $x + y = 3$; and if to this the firſt Equation be added, the Sum will be $2x = 10$, and conſequently $x = 5$.

QUESTION III.

Given the Sum of two Numbers $= 7$, and the Sum of their Squares $= 25$; what are thoſe Numbers?

Solution.

Put x for the one Number, and y for the other; then,

1 $x + y = 7$
2 $x^2 + y^2 = 25$ } Per Queſtion.

From the firſt Equation,

3 $x = 7 - y$, therefore $x^2 = 49 - 14y + y^2$. And if this Value be ſubſtituted for x^2 in the ſecond Equation, we ſhall have

4 $49 - 14y + 2y^2 = 25$; and, by Tranſpoſition and Diviſion,

5 $y^2 - 7y = -12$. Complete the Square, and there will be

6 $y^2 - 7y + \frac{49}{4} = \frac{1}{4}$, (for $-12 + \frac{49}{4} = -\frac{48}{4} + \frac{49}{4} = \frac{1}{4}$.) Extract the Square Root on both Sides of the Equation, and we ſhall have

7 $y - \frac{7}{2}{}^* = \sqrt{\frac{1}{4}} = \frac{1}{2}$. Laſtly, $\varphi - \frac{7}{2}$, and we have

8 $y = \frac{7}{2} + \frac{1}{2} = \frac{8}{2} = 4$; and then x if found to be $= 7 - 4 = 3$.

Another Solution of the laſt Queſtion.

Since $x + y = 7$, ſquare both Sides of this Equation, and we ſhall have $x^2 + 2xy + y^2 = 49$. Take the 2d Equation from it, $x^2 \quad + y^2 = 25$, and the Remainder will be $* + 2xy \quad * = 24$. If now this Rem. be taken from 2d Equat. $\}$ $x^2 \quad y^2 = 25$, there will then remain $x^2 - 2xy + y^2 = 1$. The firſt Side of this Equation is evidently a perfect Square; and therefore, by extracting the Root on both Sides, we have $x - y$ (or $y - x$) $= \sqrt{1} = 1$. We have now given both the Sum and Difference of two Quantities, to find them. Suppoſe x to be the greater, and add theſe Equations together

$$\begin{array}{rr} & x + y = 7 \\ & x - y = 1 \\ \hline \text{The Sum is} & 2x \quad * = 8 \\ \text{And} & x = 4 \end{array}$$

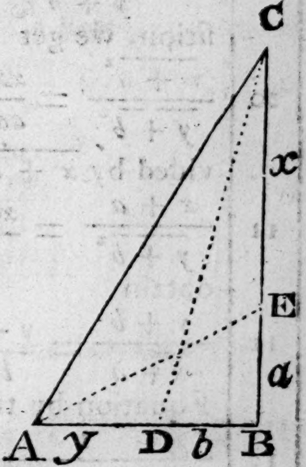
QUESTION IV.

There is a right-angled triangular Garden A, B, C, in whoſe Sides ſtand two Trees, D and E, at the Diſtances of 8 and 10 Yards, reſpectively, from the right Angle; moreover, ſtanding at the acute Angles, I obſerved the Trees appeared to ſtand in the Middle of the Sides. From this *Data* the Dimenſions of the Garden are required.

---

* The Root might have been written $\frac{7}{2} - y$. This will be explained in the next Section.

Solution.



Since to an Eye at the oppoſite Angles the Trees appeared to ſtand in the Middle of the Sides, it is evident, that, if Lines were drawn from thoſe Angles to the Trees, both the acute Angles would be biſected. This premiſed, put $a = BE$, $b = BD$, $x = EC$, and $y = DA$; then, by Euclid 3. VI. BE : EC :: AB : AC. That is,

1 $a : x :: y + b : \frac{x}{a} \times \overline{y + b} = AC$. Again, by the ſame Prop.
BD : DA :: CB : CA; *i. e.*

2 $b : y :: x + a : \frac{y}{b} \times \overline{x + a} = CA$. Conſequently we have

3 $\frac{x}{a} \times \overline{y + b} = \frac{y}{b} \times \overline{x + a}$. This reduced becomes

4 $bxy + bbx = axy + aay$; and, by Tranſpoſition,

5 $axy - bxy + aay = bbx$, or $\overline{ax - bx + aa} \times y = bbx$; and by Diviſion,

6 $y = \frac{bbx}{ax - bx + aa}$, which is $= \frac{bbx}{\overline{a - b} \times x + aa}$.

Now in order to obtain another Expreſſion of the Value of y in Terms of x and known Quantities, we muſt conſider another Property of the Figure. And ſince the Triangle is right-angled, we have, by Euclid 47. I.
$AB^2 + BC^2 = AC^2$. Therefore,

7 $\overline{y + b}^2 + \overline{x + a}^2 = \frac{xx}{aa} \times \overline{y + b}^2$, by the firſt Step. And

8 $\overline{y + b}^2 + \overline{x + a}^2 = \frac{yy}{bb} \times \overline{x + a}^2$, by the 2d Step.

Divide the 7th Equation by $\overline{y + b}^2$, and we ſhall have

9 $1 + \frac{\overline{x+a}^2}{\overline{y+b}^2} = \frac{xx}{aa}$, from which, by Tranſpoſition, we get

10 $\frac{\overline{x+a}^2}{\overline{y+b}^2} = \frac{xx}{aa} - 1 = \frac{xx-aa}{aa}$; and this divided by $\overline{x+a}$, gives

11 $\frac{x+a}{\overline{y+b}^2} = \frac{x-a}{aa}$. By a ſimilar Proceſs we obtain

12 $\frac{y+b}{\overline{x+a}^2} = \frac{y-b}{bb}$. Now multiply the 11th Equation by the 12th, and we ſhall have

13 $\frac{1}{\overline{y+b} \times \overline{x+a}} = \frac{\overline{x-a} \times \overline{y-b}}{aabb}$; and this, out of Fractions, is

14 $aabb = \overline{xx-aa} \times \overline{yy-bb}$, or, $aabb = \overline{xx-aa} \times yy - bbxx + aabb$, from which we obtain

15 $yy = \frac{bbxx}{xx-aa}$. But from the Value of y obtained in the 6th Step, we have,

16 $yy = \frac{bbbbxx}{\overline{aa-2ab+bb} \times xx + \overline{a-b} \times 2aax + aaaa}$, which muſt be $= \frac{bbxx}{xx-aa}$; *i. e.* dividing both Sides of this Equation by $bbxx$,

17 $\frac{bb}{\overline{aa-2ab+bb} \times xx + \overline{a-b} \times 2aax + aaaa} = \frac{1}{xx-aa}$; and this when brought out of Fractions, and $bbxx$ cancelled on both Sides, becomes

18 $-aabb = \overline{aa-2ab} \times xx + \overline{a-b} \times 2aax + aaaa$; and this Equation divided by a, and tranſpoſed, is

19 $\overline{2b-a} \times xx + \overline{b-a} \times 2ax = aaa + abb$. Now, divide by $\overline{2b-a}$, to get xx without a Co-Efficient, and we have

20 $xx + \frac{2ab - 2aa}{2b - a} x = \frac{aaa + abb}{2b - a}$; next complete the Square, and multiply both Numerator and Denominator of $\frac{aaa + abb}{2b - a}$ by $2b - a$, and we ſhall have

21 $xx + \frac{2ab - 2aa}{2b - a} x + \overline{\frac{ab - aa}{2b - a}}\Big|^2$
$= \frac{2aaab + 2abbb - aaaa - aabb}{4bb - 4ab + aa}$
$+ \frac{aabb - 2aaab + aaaa}{4bb - 4ab + aa}$, which is evidently
$= \frac{2abbb}{4bb - 4ab + aa}$. Then, by extracting the Square Root on both Sides, we get

22 $x + \frac{ab - aa}{2b - a} = \frac{\sqrt{2abbb}}{2b - a} = \frac{b\sqrt{2ab}}{2b - a}$; and by Tranſpoſition,

23 $x = \frac{aa - ab + b\sqrt{2ab}}{2b - a}$. In Numbers,
$x = \frac{100 - 80 + 8\sqrt{160}}{16 - 10} = \frac{20 + 8\sqrt{160}}{6}$
$= \frac{10 + 4\sqrt{160}}{3} = 20{\cdot}1988$. The Value of x being now known, that of y will be had from the 6th Step, and will be found $= 9{\cdot}2076$. And then we have

24 $AB = 17{\cdot}2076$, and $BC = 30{\cdot}1988$; and by writing the Values of x and y in the firſt Step, we have $AC = 34{\cdot}7573$.
The Dimenſions of the Garden being now known, the Area is eaſily obtained, which is

25 259·8244 Square Yards.

But this Queſtion may be anſwered by Means of Geometry and Trigonometry, without any Aſſiſtance from Algebra.

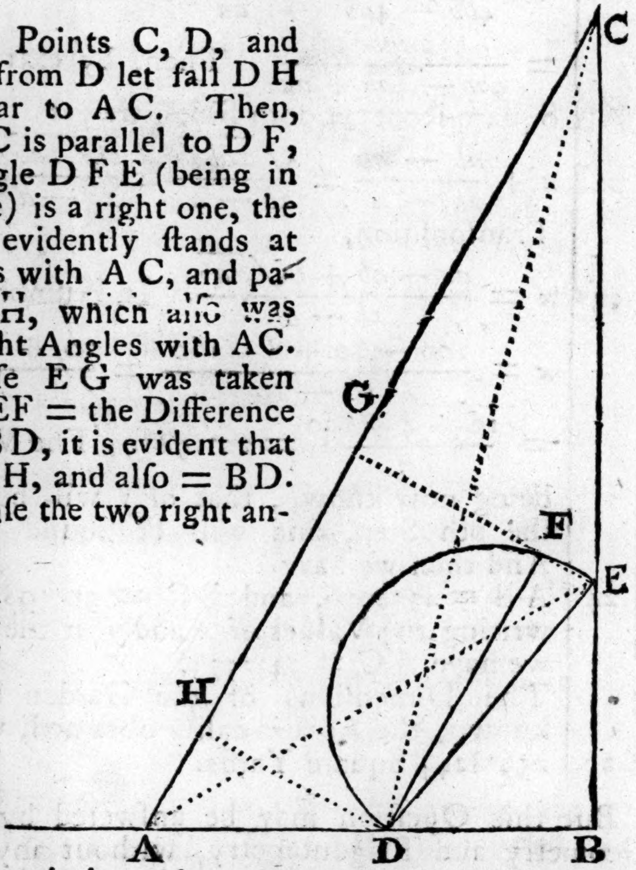
CON-

CONSTRUCTION.

With two indefinite right Lines make a right Angle at B. Take B D and B E = the reſpective Diſtances of the Trees from the right Angle. Join the Points D and E, and upon D E deſcribe a Semicircle, in which, from E, apply EF = BE — BD. Produce E F till E G is = E B. Laſtly, join the Points D and F, and parallel to D F, through G, draw a right Line, and it will cut the two indefinite right Lines that were firſt drawn in the Points A and C; and A B C will be the Triangle required.

DEMONSTRATION.

Join the Points C, D, and A, E, and from D let fall D H perpendicular to A C. Then, becauſe A C is parallel to D F, and the Angle D F E (being in a Semicircle) is a right one, the Line E G evidently ſtands at right Angles with A C, and parallel to D H, which alſo was drawn at right Angles with AC. And becauſe E G was taken = EB, and EF = the Difference of B E and B D, it is evident that F G is = D H, and alſo = B D. Now, becauſe the two right angled Triangles C D B and C D H have the ſame Line, C D, for their Hypothenuſe, and the Sides D B and D H = to each other, it is evident that their other Sides and Angles are reſpectively = each other; and conſequently that the Line C D biſects the Angle A C B. By the ſame



ſame Argument it is proved that the Line A E biſects the Angle C A B, *Q. E. D.*

Calculation.

As the Sides of the right-angled Triangle D B E are given = 8 and 10 Yards, reſpectively, the Hypothenuſe D E is $= \sqrt{164} = 2\sqrt{41} = 12{\cdot}80625$; or it might be found by Trigonometrical Calculation, by which it appears, that the Angle E D B $= 51^\circ\, 20'\, 24''\frac{1}{2}$. Again, in the right-angled Triangle D F E, there is given the Hypothenuſe D E $= 12{\cdot}8062$, and E F $= 2$; from which the Angle E D F is found $= 8^\circ\, 59'\, 5''\frac{1}{2}$. Add this Angle to the Angle E D B, and we have the Angle F D B $= 60^\circ\, 19'\frac{1}{2}$; which, becauſe D F and A C are parallel, is = the Angle C A B; and, becauſe the Triangles A B C and E G C are ſimilar, it is alſo = the Angle C E G. Then, as E G is = 10, we readily find E C $= 20{\cdot}1988$. B C being now known, as well as all the Angles of the Triangle A B C, we may eaſily calculate A B; or we may calculate A D, as all the Angles and the Side H D, of the Triangle A H D, are given; and thence the Area.

Remark 1. This Problem admits of an eaſy general Conſtruction, let the Angle at B be what it will, thus: Deſcribe a Circle through the Points D B E, and in it apply E F = B E — B D. The reſt of the Conſtruction is the ſame as before.

Remark 2. B E ought always to be to B D in a leſs Ratio than 2 to 1.

SECT. II.

Some further Obſervations on Quadratics; in which it is proved, that every ſuch Equation has two Roots.

THE Learner may have obſerved, from what has been done in the Appendix and Supplement, that Quadratic Equations have different Forms; *i. e.* that not only the ſecond Term is ſometimes poſitive and ſometimes

times negative, but that the third Term alſo has ſometimes an Affirmative, and ſometimes a negative Sign.

If the unknown Quantity be denoted by x, the Coefficient of the ſecond Term by p, and the third Term by q, all the different Forms of Quadratics will be exhibited by the following four Equations, in which the Quantities p and q are always affirmative.

$$1,\ x^2 + px = q$$
$$2.\ x^2 -- px = q$$
$$3.\ x^2 - px = - q$$
$$4.\ x^2 + px = - q.$$

The firſt Side of each of the above Equations is evidently an imperfect Square; which, however, may be rendered complete by the Addition of $\frac{pp}{4}$, which muſt be put on the other Side alſo, to preſerve the Equality. The above Equations, then, when the Square is completed, will become

$$1.\ x^2 + px + \frac{pp}{4} = q + \frac{pp}{4}$$
$$2.\ x^2 - px + \frac{pp}{4} = q + \frac{pp}{4}$$
$$3.\ x^2 - px + \frac{pp}{4} = - q + \frac{pp}{4}$$
$$4.\ x^2 + px + \frac{pp}{4} = - q + \frac{pp}{4}$$

Now, if we extract the Square Root on both Sides of each of theſe Equations, we ſhall have

$$1.\ x + \frac{p}{2} = \sqrt{q + \frac{pp}{4}}$$
$$2.\ x - \frac{p}{2} = \sqrt{q + \frac{pp}{4}}$$
$$3.\ x - \frac{p}{2} = \sqrt{- q + \frac{pp}{4}}$$
$$4.\ x + \frac{p}{2} = \sqrt{- q + \frac{pp}{4}}$$

And, laſtly, by tranſpoſing the known Quantity from the firſt Side to the ſecond, we obtain a Value of x in each of theſe Equations, thus:

$$1.\quad x = -\frac{p}{2} + \sqrt{q + \frac{pp}{2}}$$

$$2.\quad x = +\frac{p}{2} + \sqrt{q + \frac{pp}{4}}$$

$$3.\quad x = +\frac{p}{2} + \sqrt{-q + \frac{pp}{4}}$$

$$4.\quad x = -\frac{p}{2} + \sqrt{-q + \frac{pp}{4}}$$

The laſt four Equations furniſh us with the following Particulars:

In the firſt of theſe Equations the Value of x is affirmative. For as p and q are affirmative Quantities, it is evident that $\sqrt{q + \frac{pp}{4}}$ muſt be greater than $\frac{p}{2}$, and conſequently $-\frac{p}{2} + \sqrt{q + \frac{pp}{4}}$ is an affirmative Quantity.

In the 2d Equation, the Value of x is $\frac{p}{2} + \sqrt{q + \frac{pp}{4}}$ which is not only affirmative, but greater than p itſelf.

In the third Equation, the Value of x is alſo poſitive ſo long as $\sqrt{-q + \frac{pp}{4}}$ is a poſſible Quantity, but leſs than p, becauſe $\sqrt{-q + \frac{pp}{4}}$ is evidently leſs than $\frac{p}{2}$. It may ſometimes happen, that q ſhall $= \frac{pp}{4}$, in which Caſe $\sqrt{-q + \frac{pp}{4}}$ vaniſhes, or becomes $= 0$; and then x is $= \frac{p}{2}$. Or it may ſo happen, that q ſhall be even greater than $\frac{pp}{4}$, in which Caſe the Square Root of $-q+\frac{pp}{4}$ is an impoſſible Quantity, or ſuch as cannot be expreſſed but by means of the radical Sign. The ſimpleſt Way of denoting it is thus $\sqrt{-c}$, where $-c=\frac{pp}{4}-q$. This Kind of Surds ſhall be treated of in the next Section. In the fourth Equation, the Value of x is negative, becauſe $\sqrt{-q+\frac{pp}{4}}$ is leſs than $\frac{p}{2}$.

I come now to ſhow, that *every Quadratic Equation has two Roots.*

In order to this it may be proper to premiſe, that the Square Root of any Quantity, as aa, is not only a, but alſo $-a$; for each of theſe Roots multiplied by itſelf produces aa.

It therefore follows, that the Square Root of any other Quantity, as $q+\frac{pp}{4}$, is as truly expreſſed by $-\sqrt{q+\frac{pp}{4}}$, as by $+\sqrt{q+\frac{pp}{4}}$; and that, in each of the four Forms of Quadratics, x has another Value, which is expreſſed thus:

1. $x=-\frac{p}{2}-\sqrt{q+\frac{pp}{4}}$
2. $x=+\frac{p}{2}-\sqrt{q+\frac{pp}{4}}$
3. $x=+\frac{p}{2}-\sqrt{-q+\frac{pp}{4}}$
4. $x=-\frac{p}{2}-\sqrt{-q+\frac{pp}{4}}$

Now it is obvious that the Value of x in the firſt of theſe four Equations is negative. So alſo is the Value of x in the ſecond, for as much as $\sqrt{q+\frac{pp}{4}}$ is greater than $\frac{p}{2}$. But in the third Equation the Value of x is poſitive, becauſe $\sqrt{-q+\frac{pp}{4}}$ is leſs than $\frac{p}{2}$. In the fourth Equation the Value of x is evidently negative.

The Sum then of the above Obſervations is this: *That every Quadratic Equation of the firſt and ſecond Form has* one poſitive, and one negative Root; *that every Quadratic of the third Form has* two poſitive Roots, *except when q is greater than $\frac{pp}{4}$, in which Caſe its Roots are* impoſſible; *and that every Quadratic of the fourth Form has* two negative Roots, *except when q is greater than $\frac{pp}{4}$, in which Caſe they alſo are impoſſible.*

I will give an Example of each of the four Forms of Quadratics, in the Order they ſtand.

$$
\begin{aligned}
&1.\ x^2 + 4x = 21\\
&2.\ x^2 - 2x = 48\\
&3.\ x^2 - 6x = -8\\
&4.\ x^3 + 12x = -32
\end{aligned}
$$

By compleating the Square, &c. we ſhall obtain the Values of x in each of theſe Equations, in the Order they ſtand, thus:

$$1.\ x = \left\{\begin{matrix} -2+\sqrt{25} \\ \text{or} \\ -2-\sqrt{25} \end{matrix}\right\}\ i.\,e.\ \left\{\begin{matrix} +3 \\ \\ -7 \end{matrix}\right.$$

$$2.\ x = \left\{\begin{matrix} +1+\sqrt{49} \\ \text{or} \\ +1-\sqrt{49} \end{matrix}\right\}\ i.\,e.\ \left\{\begin{matrix} +8 \\ \\ -6 \end{matrix}\right.$$

3.

$$3.\ x = \left\{\begin{matrix} +3+\sqrt{1} \\ \text{or} \\ +3-\sqrt{1} \end{matrix}\right\}\ i.\ e.\ \left\{\begin{matrix} +4 \\ \\ +2 \end{matrix}\right.$$

$$4.\ x = \left\{\begin{matrix} -6+\sqrt{4} \\ \text{or} \\ -6-\sqrt{4} \end{matrix}\right\}\ i.\ e.\ \left\{\begin{matrix} -4 \\ \\ -8 \end{matrix}\right.$$

And that either of the two Values of x will be admitted into the original Equation is evident; for Inſtance, + 3 being written for x in the firſt Equation, it becomes $9 + 12$, which is $= 21$. Again, -7 being written in it for x, we have $49 - 28$, which is alſo $= 21$. And ſo of the reſt, which the Learner may try at his Leiſure.

We may add this general Remark, with reſpect to the Roots of Quadratic Equations, that the negative Values of x in Forms 1, 2, and 4, cannot anſwer the Conditions of a Queſtion that requires a poſitive Number or Quantity; and that it frequently happens, that each of the affirmative Values of x in the third Form will anſwer all the Conditions of the Queſtion, on which Account it is called the ambiguous Quadratic, and the Values of x are commonly expreſſed thus: $x = \frac{p}{2} \pm \sqrt{-q + \frac{pp}{4}}$.

There is indeed another Way of demonſtrating, that every Quadratic has two Roots; but as that given above ſeemed beſt adapted to this Work, I have for that Reaſon made Choice of it.

Mr. Ward, in his *Young Mathematician's Guide*, Part II. Chap. 8, gives a Method of reducing adfected Quadratics to ſimple Quadratics, by a Subſtitution that deſtroys the ſecond Term; but he juſtly obſerves, that the eaſieſt Solution of Quadratics is by completing the Square. They who would ſee this Method may conſult the Book before mentioned, or Mr. *Maclaurin*'s Algebra.

I ſhall only add, under this Head, that

$$x^4 + px^2 = q,$$

and $x^6 + px^3 = q$, are called Quadratics. For, by completing the Square, &c. we get

$$x^2 = -\frac{p}{2} + \sqrt{q + \frac{pp}{4}}$$

and $x^3 = -\frac{p}{2} + \sqrt{q + \frac{pp}{4}}$; and then, by extracting the Square Root of the one, and the Cube Root of the other, we obtain

$x = \sqrt{-\frac{p}{2} + \sqrt{q + \frac{pp}{4}}}$ in the former Equation,

and $x = \sqrt[3]{-\frac{p}{2} + \sqrt{q + \frac{pp}{4}}}$ in the latter.

SECT. III.

Of Surd Quantities.

THE Doctrine of Surds is of conſiderable Uſe in Algebra, and, if fully treated of, would make a Volume of itſelf. I hope, however, in the few Pages allotted me, to give the Learner ſuch a Notion of Surd Quantities as may enable him to ſhorten many Calculations in which they occur, and induce him to conſult fuller Treatiſes on the Subject, which he may find in Mr. *Maclaurin*'s and Dr. *Saunderſon*'s Algebra.

Dr. *Saunderſon*'s Definitions of Surd Quantities are as follows:

“ *A rational Number is that, whoſe Ratio or Propor-*
“ *tion to Unity can be expreſſed in finite Numbers; and*
“ *ſo comprehends all whole Numbers, mixt Numbers, and*
“ *proper Fractions:* thus, $2\frac{3}{4}$ is a rational Number, be-
“ cauſe $2\frac{3}{4}$ is to one as 11 to 4. *Therefore an irrational*
“ *Number is that whoſe Ratio to Unity cannot be expreſſed*
“ *in finite Numbers:* Thus if the Diameter of a Circle
“ be repreſented by Unity or 1, the Circumference

“ muſt be repreſented by an irrational Number; becauſe
“ the Ratio of the Circumference to the Diameter
“ cannot be expreſſed by finite Numbers: Thus,
“ $\sqrt{2}$ is an irrational Number, becauſe no Numbers
“ are ſubtil enough to expreſs its Ratio to Unity.
“ *Now this laſt irrational Number, to wit* $\sqrt{2}$, *and all*
“ *others involving the Roots of Numbers that are not to be*
“ *extracted, are particularly called Surds.*”

Now, although $\sqrt{2}$ cannot be exactly expreſſed in Numbers, yet it may be expreſſed near enough for any Buſineſs whatever, and ſo near the Truth as not to err a Hair's Breadth in ſo great a Diſtance as that of the Earth from the fixed Stars.

And from the Square Root of 2, expreſſed by a ſufficient Number of Decimals, we may find the Square Roots of many other Numbers by Multiplication only; as 8, 18, 32, 50, &c. And generally, from $\sqrt{b}$ we may find $\sqrt{aab}$, ſo long as a is a rational Quantity, by Multiplication only, as it will be $= a\sqrt{b}$. And that this is the true Root of $\sqrt{aab}$, is evident by Involution. For $a\sqrt{b}$ multiplied by itſelf produces $aa \times b$.

Now, ſince 8 is $= 4 \times 2$, $\sqrt{8}$ will be $= 2\sqrt{2}$. This may be illuſtrated by Decimals, thus: $\sqrt{2}$ is known to be $= 1{\cdot}41421$ nearly; therefore $2\sqrt{2} = 2{\cdot}82842$ nearly. And $\sqrt{8}$, in Decimals, is alſo $= 2{\cdot}82842$ nearly.

Again, 18 is $= 9 \times 2$; and therefore $\sqrt{18} = 3\sqrt{2}$.

In like Manner, $\sqrt{32}$ is found $= 4\sqrt{2}$; and $\sqrt{50} = 5\sqrt{2}$.

Alſo from $\sqrt{3}$ we get the Square Roots of 12, 27, 48, 75, &c. by Multiplication only, which Roots are $2\sqrt{3}$, $3\sqrt{3}$, $4\sqrt{3}$, $5\sqrt{3}$, &c.

With reſpect to the Reciprocal of any of theſe Surds, it is evident to an Arithmetician, that it may be found by dividing 1 by the Number which expreſſes the Value of that Surd: Thus, $\frac{1}{\sqrt{2}}$ is $= \frac{1}{1{\cdot}41421} = 0{\cdot}70710$. But this Decimal may be obtained much eaſier, as follows: Multiply both the Numerator and Denominator of $\frac{1}{\sqrt{2}}$ by $\sqrt{2}$, and it becomes $\frac{\sqrt{2}}{2} = \frac{1{\cdot}41421}{2}$ $= 0{\cdot}70710$.

In like Manner, the Square Root of $\frac{1}{3}$, which is $= \frac{1}{\sqrt{3}}$, is alſo $= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1{\cdot}73205}{3} = 0{\cdot}57735$; where the Diviſion of 1·73205 by 3 is undoubtedly an eaſier Operation than the Diviſion of 1 by 1·73205.

There are yet other Advantages of Calculation to be derived from the Knowledge of Surds.

The Square Root of $\frac{2}{3}$ is $= \frac{\sqrt{2}}{\sqrt{3}}$, or, in Decimals, $= \sqrt{0{\cdot}666666}$, &c. But the Value of this Surd may be had from $\sqrt{6}$ by an eaſy Diviſion, thus: Multiply both Numerator and Denominator of $\frac{2}{3}$ by 3, and we have $\frac{2}{3} = \frac{6}{9}$; then extract the Square Roots, and we have $\sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$.

Again the Square Root of $\frac{3}{5}$ is $= \frac{\sqrt{3}}{\sqrt{5}}$; but by multiplying both Numerator and Denominator by 20, we have $\frac{3}{5} = \frac{60}{100}$, and $\sqrt{\frac{3}{5}} = \frac{\sqrt{60}}{10}$; where, when the Square Root of 60 is extracted to a ſufficient Number of Places of Decimals, we have only to remove the Point one Place towards the Left Hand.

It will ſometimes be of Uſe to know quickly, and without the Trouble of extracting the Roots, which of two Surds is the greater, as $\sqrt{2}$, and $\sqrt[3]{3}$. This may be done by raiſing the Roots to equal Powers that are free from Surds, and comparing thoſe Powers with each other. And the leaſt equal Powers that are clear of Surds may be found by this

General Rule.

Divide the Product of the Indices of the two Roots by their greateſt common Meaſure; and the Quotient ſhall be the Index of the Power to which both Roots are to be involved for the Compariſon before mentioned.

Let us now apply this Rule to find whether $\sqrt{2}$, or $\sqrt[3]{3}$ be the greater. Here the Product of the Indices is $2 \times 3 = 6$; and their greateſt common Meaſure is 1:

conſequently $6 \div 1$ is $= 6$, the Power to which both theſe Surd Roots are to be raiſed for the Compariſon. But this Matter may be illuſtrated, and ſome uſeful Concluſions obtained, as follows: Put $x = \sqrt{2}$, and $y = \sqrt[3]{3}$; then will $x^2 = 2$, and $y^3 = 3$; and $x^6 = \overline{2}|^3 = 8$, and $y^6 = \overline{3}|^2 = 9$. Now, ſince x^6 is $= 8$, and $y^6 = 9$, it is evident that y is the greater Quantity. And if we extract equal Roots we ſhall obtain this Concluſion, $\sqrt{2}\ (= x) = \sqrt[6]{8}$; and $\sqrt[3]{3}\ (= y) = \sqrt[6]{9}$.

To find whether $\sqrt{3}$, or $\sqrt[4]{5}$ be the greater, we have $2 \times 4 = 8$, the Product of the Indices, and $8 \div 2$ (their greateſt common Meaſure) $= 4$, the Index of the Power to which theſe Surds are to be raiſed for Compariſon; from which it will appear, that the former of them is much greater than the latter, being $= \sqrt[4]{9}$.

If it were required to expreſs the Ratio of $\sqrt[3]{3}$ to $\sqrt[4]{6}$ by Means of two other Surds of equal Indices, it might be done thus: Having, by proceeding as above, found that $\sqrt[3]{3} = \sqrt[12]{81}$, and $\sqrt[4]{6} = \sqrt[12]{216}$; reſolve each of the Numbers 81 and 216 into two Factors by Means of their greateſt common Meaſure 27, and we ſhall have $81 = 3 \times 27$, and $216 = 8 \times 27$; and then, by extracting equal Roots, $\sqrt[12]{81} = \sqrt[12]{3} \times \sqrt[12]{27}$, and $\sqrt[12]{216} = \sqrt[12]{8} \times \sqrt[12]{27}$, and the Proportion will be, as $\sqrt[3]{3} : \sqrt[4]{6}$ $(:: \sqrt[12]{81} : \sqrt[12]{216}) :: \sqrt[12]{3} : \sqrt[12]{8}$.

The laſt Advantage that I ſhall mention, in calculating the Value of Surds, is, that the Square Roots of ſome compound Surds may be expreſſed by ſimple ones.

For Example, $\sqrt{3 + \sqrt{8}}$ is $= 1 + \sqrt{2}$; where, inſtead of two Extractions of the Square Root, firſt of 8, which comes out $= 2{\cdot}82842$, and then of $5{\cdot}82842$, which is $2{\cdot}41421$, we need only extract the Square Root of 2 and add 1 to it, and we ſhall have $1 + 1{\cdot}41421 = 2{\cdot}41421$.

The Inveſtigation of the above Root is as follows:

1 Put $\sqrt{x} + \sqrt{y} = \sqrt{3 + \sqrt{8}}$; then by ſquaring both Sides of the Equation,
2 $x + 2\sqrt{x}\sqrt{y} + y = 3 + \sqrt{8}$; and by putting the Surd on one Side = the Surd on the other,
3 $2\sqrt{x}\sqrt{y} = \sqrt{8}$,
4 $x + y = 3$. Square 3d and 4th Equations, and we have
5 $4xy = 8$, and
6 $x^2 + 2xy + y^2 = 9$. Take the 5th from the 6th and we ſhall have
7 $x^2 - 2xy + y^2 = 1$, which is a perfect Square. Extract the Root, and there will be
8 $x - y = \sqrt{1} = 1$. Add this to the 4th Equation, and the Sum is
9 $2x = 4$; from which $x = 2$, and
10 $\sqrt{x} = \sqrt{2}$, which is one Part of the Root. And if the 8th Equation be taken from the 4th, the Remainder will be
11 $2y = 2$; from which y is found $= 1$, and
12 $\sqrt{y} = \sqrt{1} = 1$, which is the other Part of the Root.

Thoſe who would ſee more of this, I refer to the Books above mentioned; and ſhall only add, that we ſometimes meet with Surds of this Form, $\sqrt{-c}$, which is called an impoſſible Root, becauſe no Quantity whatever multiplied by itſelf will produce $-c$. For Inſtance, let $-c = -4$; then there is no Number whatever, which, being multiplied by itſelf, will produce -4. For $+2 \times +2$ produces $+4$; and -2×-2 produces $= +4$.

SECTION IV.

Some further Obſervations on the Extraction of Roots.

THE Extractions of the Roots of Numbers are Operations frequently requiſite in the Practice of the Mathematics, and, on that Account, have engaged the Attention of the moſt eminent Mathematicians. And to their Ingenuity we are indebted for Theorems, by which we may quickly approximate to the Value of any Root whatever of any Number; or, in other Words, they have invented Rules by which any Root whatever of any Number may be expreſſed to any Degree of Exactneſs, either in Vulgar Fractions or Decimals.

Of theſe Theorems I will here give thoſe, which, conſidering their Facility and Accuracy, are the moſt generally uſeful. They are of two Kinds, one that requires no higher Operation in Arithmetic than Diviſion, and is therefore called *rational*; the other requires the Extraction of the Square Root, and is therefore called *irrational*.

I will firſt explain the *rational Theorems*, beginning with the *Square Root*, and ending with the *Surſolid*.

Let N be the Number whoſe Square Root is to be extracted, aa the next leſs Square, and put b for the Difference of this Square and N, *i. e.* let $aa + b = N$; let alſo $a + e = \sqrt{N}$; then by ſquaring both Sides of this Equation we get

1 $aa + 2ae + ee = N = aa + b$; and by cancelling aa on both Sides, we have

2 $2ae + ee = b$, or $e \times \overline{2a + e} = b$; in which Equation, becauſe e is ſmall in Compariſon of $2a$, it may be neglected without making any conſiderable Difference in the Equation, and there will be $e \times 2a = b$ nearly, and

3 $e = \frac{b}{2a}$ nearly; which Value of e is evidently ſomewhat too great, the true Value being $\frac{b}{2a + e}$. Now, ſince $\frac{b}{2a}$ is a near Value of e, if that be written for e in the Diviſor $2a + e$, we ſhall have

4 $e = \frac{b}{2a + e} = \frac{b}{2a + \frac{b}{2a}} =$ (by multiplying both Numerator and Denominator by $2a$) $\frac{2ab}{4aa + b}$, which is a much nearer Value of e than the former.

But ſince $\frac{b}{2a}$ is ſomewhat greater than e, it is evident that the Diviſor $2a + \frac{b}{2a}$ is ſomewhat greater than $2a + e$, and conſequently, that the laſt found Value of e is ſomewhat too little.

5 Laſtly, by writing $\frac{2ab}{4aa + b}$ for e in the Root, we have $a + e = \sqrt{N} = a + \frac{2ab}{4aa + b}$ nearly, which is the Theorem.

And although the Value of the Root given by the Right Hand Side of the laſt Equation, is always ſomewhat too little, yet, by Means of this Theorem, the Value of it, true to any aſſigned Number of Figures, may be quickly diſcovered.

Note, It will ſometimes happen, that the Square Number neareſt in Value to N, ſhall be greater than N. In which Caſe, if we put $aa - b = N$, and $a - e = \sqrt{N}$, and proceed as above, we ſhall get $e = \frac{b}{2a - e}$; where, at firſt rejecting e, in the Diviſor, on Account of its Smallneſs, we have $e = \frac{b}{2a}$ nearly, which Value is evidently too ſmall; and by ſubſtituting this Value of e for it in the Diviſor, we obtain $e = \frac{b}{2a - e} = \frac{b}{2a - \frac{b}{2a}} = \frac{2ab}{4aa - b}$, which is a much nearer Value of e, but ſtill ſomewhat too little; becauſe $\frac{b}{2a}$ being leſs than e, the Diviſor $2a - \frac{b}{2a}$ is greater than $2a - e$. But if the laſt found Value of e be written for it in the above Aſſumption, we ſhall have $a - e = \sqrt{N} = a - \frac{ab}{4aa - b}$ nearly; which Value, however, of $\sqrt{N}$, will always be ſomething too great.

We may here obſerve, that this Theorem differs from the firſt only in the Sign of b.

The Uſe of theſe Theorems will be beſt explained by Examples.

EXAMPLE I.

Let it be required to expreſs the Square Root of 2, both in Vulgar Fractions and Decimals, ſo near the Truth that it ſhall not err the 1000th Part of an Unit.

Here the neareſt leſs Square Number is 1; and therefore $aa + b = N$, in this Caſe, becomes $1 + 1 = 2$, and $\surd 2$ is nearly $= 1 + \frac{2}{4+1} = 1\frac{2}{5} = \frac{7}{5} = 1{\cdot}4$, which is ſomewhat too ſmall. Now, to get a nearer Value of $\surd 2$, put $\frac{7}{5}$ for a new a; then, $aa = \frac{49}{25}$; and by putting $\frac{1}{25}$ for a new b, we ſhall have $\frac{49}{25} + \frac{1}{25} = 2$, and $\surd 2 = \frac{7}{5} + \frac{14}{5} \times \frac{1}{25} \div \overline{\frac{196}{25} + \frac{1}{25}} = \frac{7}{5} + \frac{14}{5 \times 197} = \frac{1393}{985} = 1\frac{408}{985}$ nearly, which, however, is ſtill ſomewhat too ſmall. This Value of $\surd 2$, in Decimals, is $1{\cdot}4142132$.

It will now be proper to aſcertain how near theſe Expreſſions, $1\frac{408}{985}$, and $1{\cdot}4142132$ are to the true Value of $\surd 2$. This may be done ſeveral Ways, but by none more ſatisfactorily than by the other Theorem, thus: Take the next greater Square, which is 4, for the Square of a; then ſhall we have $a = 2$, and $b = 2$; and $aa - b = N$, in the preſent Caſe, will be $4 - 2 = 2$; from which we have $\surd 2$ nearly $= 2 - \frac{4 \times 2}{16 - 2} = 2 - \frac{4}{7} = \frac{10}{7} = 1{\cdot}428$, which is ſomewhat too great: But if $\frac{10}{7}$ be put for a new a, the Value of b correſponding to it will appear to be $\frac{2}{49}$, as we ſhall have $\frac{100}{49} - \frac{2}{49} = 2$; and then $\surd 2$ will be very nearly $= \frac{10}{7} - \frac{20}{7} \times \frac{2}{49} \div \overline{\frac{400}{49} - \frac{2}{49}} = \frac{10}{7} - \frac{40}{7 \times 398} = \frac{3940}{2786} = \frac{1970}{1393} = 1\frac{577}{1393}$; which Number reduced to Decimals is $1{\cdot}4142139$, and, as was proved in the Inveſtigation of the Theorem, ſomewhat too great.

Now, ſince $\surd 2$ is greater than $1{\cdot}4142132$, and leſs than $1{\cdot}4142139$, it is evident that the firſt ſeven Figures of each of theſe Decimal Values of it are right, and that the $\sqrt{2}$ is expreſſed by them, as well as by the Fractions from which they were derived, to leſs than the millionth Part of an Unit.

Here one cannot but obſerve how expeditiouſly and exactly the Values of Roots may be computed by Means of theſe Theorems, even in the moſt diſadvantageous Caſes. The Square Root of 2 was diſcovered above, by each of the Theorems, by two Operations only, true to ſeven Places of Figures; and were either $\frac{1393}{985}$, or $\frac{1970}{1393}$, to be put for a, and the Operation renewed, the Reſult would be true to about twenty Places of Figures.

But it may be further obſerved, that the above Expreſſions of $\sqrt{2}$ are much more exact than was at firſt propoſed; and that Calculations made to the 1000th Part of an Unit are exact enough for many Purpoſes. In anſwer to this, no more Figures of the Decimal need be uſed than Occaſion requires: Or, if a Vulgar Fraction be deſired, of fewer Figures than either of thoſe above, but yet ſuch as ſhall not err the 1000th Part of an Unit, it may be quickly found as follows: It appears from the Inveſtigation in Page 246, that, when e is ſmall in Compariſon of a, it will be $= \frac{b}{2a}$ nearly, and conſequently $\sqrt{N}, = a + e$, will be $= a + \frac{b}{2a}$ nearly. Now, it being diſcovered that $\sqrt{2}$ is ſomewhat greater than $\frac{7}{5}$, put this Fraction $= a$, and we ſhall have $a + \frac{b}{2a} = \frac{7}{5} + \frac{1}{25} \div \frac{14}{5} = \frac{7}{5} + \frac{1}{70} = \frac{99}{70}$, which is a near Value of $\sqrt{2}$, and, if expreſſed decimally, will be $= 1{\cdot}4143$, which differs by leſs than 1000th Part of 1 from the true Value of $\sqrt{2}$. The ſame Value of it may be found by putting $a = \frac{10}{7}$, and uſing the other Theorem.

EXAMPLE 2.

Let it be required to find the Square Root of 8.

Here the neareſt Square is 9; ſo that, by Theorem II, $a = 3$, and $b = 1$, and $\sqrt{8} = 3 - \frac{6}{36 - 1} = 2\frac{29}{35}$ =

$= 2{\cdot}8486$ nearly, which is true to leſs than 1000th of an Unit. And if $\frac{99}{35}$ ($= 2\frac{29}{35}$) were put for a new a, and the Operation repeated, the Reſult would be true to twelve Places of Decimals.

EXAMPLE 3.

Let it be required to extract the Square Root of 101.

Here the neareſt Square is 100; and we have $a = 10$, $b = 1$, and $\surd 101 = 10 + \frac{20}{401} = 10{\cdot}049875$, all which Figures are true; and conſequently, the Square Root of 101 is expreſſed by them true to leſs than the millionth Part of an Unit.—And here it may be noted, that if we had taken only $a + \dfrac{b}{2a}$ for the Root, we ſhould have had $10\frac{1}{20} = 10{\cdot}05$, which differs from the Truth but little more than 10000th Part of 1, and leſs than the 800000th Part of the Whole.

I ſhall at preſent diſmiſs this Article with a Remark, that a nearer Value of $\surd N$ may be obtained by the firſt Operation, when Occaſion requires it, from this Theorem, $\surd N = a + \dfrac{b}{2a} \times \dfrac{4aa + b}{4aa + 2b}$; in which, if a was taken = the Root of the next greater Square, b will be Negative, and muſt have the Sign − prefixed to it. To give an Example; $\surd 2$, if we take $a = 1$, will be given by this Theorem, at the firſt Operation,

$$= 1 + \frac{1}{2} \times \frac{4 + 1}{4 + 2} = 1 + \frac{5}{12} = \frac{17}{12};$$

in Decimals $= 1{\cdot}4166$ nearly, which differs from the Truth by leſs than the 400th Part of an Unit. And if we take $a = 2$, then b will be $= -2$, and the Theorem gives

$$\surd 2 = 2 - \frac{2}{4} \times \frac{16 - 2}{16 - 4} = 2 - \frac{1}{2} \times \frac{14}{12} = 2 - \frac{7}{12} = \frac{17}{12},$$

as before. And were the Operation to be repeated, taking $a = \frac{17}{12}$, the Reſult would be true to twelve Places of Decimals

Of

Of the CUBE ROOT.

Let N be the Number whoſe Cube Root is to be extracted, a^3 the neareſt leſs Cube, and b the Difference, *i. e.* let $a^3 + b = N$; and put $a + e = \sqrt[3]{N}$: Then, by Involution, $a^3 + 3a^2 e + 3ae^2 + e^3 = N = a^3 + b$, and $3a^2 e + 3ae^3 + e^3 = b$; and rejecting e on account of its Smallneſs, we have $3a^2e + 3ae^2 = b$ nearly, or $e \times \overline{3a^2 + 3ae} = b$ nearly, and $e = \frac{b}{3a^2 + 3ae}$ nearly; where it is eaſy to ſee, that, when e is ſmall in Compariſon of a, $3ae$ will be ſmall in Compariſon of $3aa$; and therefore if $3ae$ be neglected in the Diviſor, we ſhall have $e = \frac{b}{3aa}$ nearly. Now, if this Value of e be written for it in the Term $3ae$, the Diviſor will become $3aa + \frac{3ab}{3aa} = 3aa + \frac{b}{a}$; and uſing this inſtead of the former, we get $e = \frac{ab}{3a^3 + b}$; and then, by Subſtitution, we have $a + e = \sqrt[3]{N} = a + \frac{ab}{3a^3 + b}$ nearly.

But as it will be moſt commodious to uſe the neareſt Cube Number, and that may ſometimes be greater than N, we muſt, in that Caſe, put $a^3 - b = N$, and $a - e = \sqrt[3]{N}$; and then, by proceeding as above, we ſhall get $-3a^2e + 3ae^2 - e^3 = -b$; and rejecting e^3 on Account of its Smallneſs, and changing the Signs, (which is equivalent to tranſpoſing the Terms) we have $3a^2e - 3ae^2 = b$; and firſt taking $e = \frac{b}{3aa}$ nearly, and then writing this Value for it in the ſecond Term of the Diviſor, we ſhall have $e = \frac{ab}{3a^3 - b}$ nearly, and conſequently $a - e = \sqrt{N} = a - \frac{ab}{3a^3 - b}$ nearly.

EXAMPLE 1.

Let it be required to extract the Cube Root of 2.

Here the neareſt Cube is 1; therefore we have $a = 1$, and $b = 1$, and $\sqrt[3]{2} = 1 + \frac{1}{3 + 1} = 1\frac{1}{4} = 1{\cdot}25$. But to get a nearer Value of this Root, put $\frac{5}{4}$ $(= 1\frac{1}{4}) = a$; then will $b = 2 - \frac{125}{64} = \frac{3}{64}$, and $\sqrt[3]{2} = \frac{5}{4} + \frac{5 \times 3}{4 \times 375 + 3} = \frac{5}{4} + \frac{5}{4 \times 125 + 1} = \frac{5}{4} + \frac{5}{504} = \frac{635}{504} = 1{\cdot}259921$ nearly. And were the Operation to be repeated, taking $a = \frac{635}{504}$, the Reſult would be true to many more Places of Figures.

EXAMPLE 2.

To find the Cube Root of 980.

The neareſt Cube is 1000, and $1000 - 980 = 20$; therefore by the ſecond Theorem, $\sqrt[3]{980} = 10 - \frac{200}{3000 - 20} = 10 - \frac{10}{149} = 9\frac{139}{149} = 9{\cdot}93288$, all which Figures are true. And by taking $a = \frac{1480}{149}$ $(= 9\frac{139}{149})$ and repeating the Operation, the Root may be computed to eighteen Places of Figures.

Of the BIQUADRATE ROOT.

The Biquadrate or fourth Root of any Number may be found by two Extractions of the Square Root; but it may be found more expeditiouſly at once, by the following Theorems.

Put N for the Number, a for the neareſt leſs Root, let $a^4 + b = N$, and $a + e = \sqrt[4]{N}$: Then, by Involution, $a^4 + 4a^3e + 6a^2e^2 + 4ae^3 + e^4 = N, = a^4 + b$; and rejecting the Terms $4ae^3$ and e^4, as being of ſmall Value in Compariſon of the reſt, we have $4a^3e + 6a^2e^2 = b$ nearly; and then, by a Proceſs ſimilar to that for the Cube Root, we get $e = \frac{2ab}{8a^4 + 3b}$ nearly, and $\sqrt[4]{N} = a + \frac{2ab}{8a^4 + 3b}$ nearly.

But when a is taken = the neareſt greater Root, b will be negative, and the Theorem will become $\sqrt[4]{N} = a - \frac{2ab}{8a^4 - 3b}$ nearly.

EXAMPLE.

What is the Biquadrate Root of 17?

The neareſt Root (in whole Numbers) is 2, and, by the firſt Theorem, we have $\sqrt[4]{17} = 2 + \frac{4}{131}$ nearly, $= 2{\cdot}03053$, which may be exact enough for common Uſe; but if greater Accuracy be deſired, it may be obtained by taking $a = \frac{266}{131}$, and renewing the Operation.

Of the SURSOLID ROOT.

By a Notation and Proceſs ſimilar to thoſe which have been all along uſed in this Section, we ſhall have $a^5 \pm b = N$, $a \pm e = \sqrt[5]{N}$, and $\pm 5a^4e + 10a^3e^2 = \pm b$ nearly; and then $e = \frac{ab}{5a^5 \pm 2b}$ nearly, and $\sqrt[5]{N} = a \pm \frac{ab}{5a^5 \pm 2b}$ nearly; where the upper of the double Signs is to be uſed when a is taken leſs than the true Root, and the lower when it is taken greater.

EXAMPLE.

What is the 5th Root of 99990?

The neareſt Root (in whole Numbers) is 10, which is ſomewhat too great; therefore we have $\sqrt[5]{99990} = 10 - \frac{100}{500000 - 20} = 10 - \frac{10}{49998} = 9\frac{24904}{24999} = 9{\cdot}9979999$ nearly.

Of the IRRATIONAL THEOREMS.

The *irrational Theorems* were treated of, by the Author, in the Appendix. However, they may be inveſtigated in a Manner ſomewhat different, and given in a

Form better adapted to Practice, as will preſently appear.

If we look back to the Inveſtigation of the *rational Theorems* for the Cube Root, we ſhall find, in Page 251, this Equation, $3aae + 3aee = b$ nearly; and dividing by $3a$, we have this Quadratic, $ee + ae = \frac{b}{3a}$ nearly; from which, comp. □, &c. we get $e + \frac{a}{2} = \sqrt{\frac{b}{3a} + \frac{aa}{4}}$; and then, by adding $\frac{a}{2}$ to both Sides, we have $a + e \; (= \sqrt[3]{N})$ $= \frac{a}{2} + \sqrt{\frac{b}{3a} + \frac{aa}{4}}$ nearly, which is the Theorem when a is taken leſs than the true Root.

But when a is taken greater than the true Root, we have this Equation, $-3aae + 3aee = -b$ nearly; from which, by comp. □, &c. we obtain $a - e \; (= \sqrt[3]{N})$ $= \frac{a}{2} + \sqrt{\frac{-b}{3a} + \frac{aa}{4}}$ nearly, which is the Theorem in this Caſe.

And in like Manner, by extracting the Roots of the Quadratic Equations in Pages 252 and 253, we obtain the following Theorems for the Biquadratic and Surſolid Roots. And ſo on for other Roots.

When a is taken leſs than the true Root.

$$\sqrt[4]{N} = \tfrac{2}{3}a + \sqrt{\frac{b}{6aa} + \frac{aa}{9}} \text{ nearly.}$$

$$\sqrt[5]{N} = \tfrac{3}{4}a + \sqrt{\frac{b}{10a^3} + \frac{aa}{16}} \text{ nearly.}$$

When a is taken greater than the true Root.

$$\sqrt[4]{N} = \tfrac{2}{3}a + \sqrt{\frac{-b}{6aa} + \frac{aa}{9}} \text{ nearly.}$$

$$\sqrt[5]{N} = \tfrac{3}{4}a + \sqrt{\frac{-b}{10a^3} + \frac{aa}{16}} \text{ nearly.}$$

That theſe Theorems are the ſame in Effect as Mr. Fenning's, is evident enough from their Inveſtigation. But it may be proved in a different Manner, of which I will give an Example in comparing his irrational Theorem for the Cube Root with ours.

Mr. Fenning's b is $=$ our N, and his x is $=$ our a; therefore, his $\frac{b}{3x} - \frac{xx}{12} =$ our $\frac{N}{3a} - \frac{aa}{12}$. Now if the laſt Expreſſion be reduced to a common Denominator, it will be $\frac{4N - a^3}{12a}$, which, becauſe $N = a^3 \pm b$, is

$$= \frac{4a^3 \pm 4b - a^3}{12a} = \frac{3a^3 \pm 4b}{12a} = \frac{3a^3}{12a} \pm \frac{4b}{12a} = \frac{aa}{4} \pm \frac{b}{3a}$$

which is the Expreſſion in our Theorem.

And in this Manner may the Agreement of the reſt of his Theorems with ours be ſhown.

Note, It appears by the Compariſon juſt now made, that $\sqrt[3]{N}$ is alſo $= \frac{1}{2}a + \sqrt{\frac{4N - a^3}{12a}}$ nearly, which is the ſame Theorem in another Form. However it may be of Uſe on ſome Occaſions.

But there is yet another Theorem for the Cube Root, eaſy in Practice, and more exact than any of thoſe above given, which I will take this Opportunity to publiſh.

$\sqrt[3]{N} = a + \frac{b}{3aa} \times \frac{9aaa + 2b}{9aaa + 5b}$ very nearly; where, if a was taken greater than the true Root, the Sign of b muſt be changed.

EXAMPLE.

What is the Cube Root of 980?

By our ſecond irrational Theorem, taking $a = 10$, we have $\sqrt[3]{980} = 5 + \sqrt{\frac{-20}{30} + \frac{100}{4}} = 5 + \sqrt{24\frac{1}{3}}$ $= 9{\cdot}932883$.

By the new Theorem, taking $a = 10$, we have $\sqrt[3]{980}$ $= 10 - \frac{20}{300} \times \frac{9000 - 40}{9000 - 100} = 10 - \frac{2}{30} \times \frac{896}{890}$ $= 9{\cdot}932883895$.

They who would ſee more on this Subject, are referred to Dr. *Halley*'s Paper, which is printed at the End of Sir *Iſaac Newton*'s *Univerſal Arithmetic*; *Maclaurin*'s *Algebra*; or *Simpſon*'s *Diſſertations*. The Subject is alſo very elegantly treated of by Dr. *Hutton*, in his *Mathematical Tracts*, lately publiſhed.

Of the EXTRACTION *of* ROOTS *by* LOGARITHMS.

Of all the Methods of extracting the Square, Cube, Biquadrate, &c. Roots of Numbers, that by Logarithms is the eaſieſt; and this is the general

RULE.

Divide the Logarithm of the Number whoſe Root is to be extracted by the Index of the Root, and the Quotient ſhall be the Logarithm of the Root.

EXAMPLE 1.

What is the Square Root of 2?

The Logarithm of 2 is $= 0{\cdot}3010300$;

The Half of it is $= 0{\cdot}1505150$; and the Number anſwering to it is 1·414214, which is the Square Root of 2.

EXAMPLE 2.

What is the Square Root of 101?

The Logarithm of 101 is $= 2{\cdot}0043214$

the Half of it is $= 1{\cdot}0021607$; and the Number anſwering to it is 10·04988, the Root required.

EXAMPLE 3.

What is the Square Root of $2\frac{1}{6}$?

The Number reduced to an improper Fraction is $\frac{13}{6}$; and then the Operation by Logarithms will ſtand thus:

The Log. of 13 is $= 1{\cdot}1139434$
The Log. of 6 is $= 0{\cdot}7781513$

Their Diff. is Log. of $\frac{13}{6} = 0{\cdot}3357921$

$\frac{1}{2}$ the laſt Log. is $= 0{\cdot}1678960$; and the Number correſponding to it is 1·47196, which is $= \sqrt{\frac{13}{6}}$.

Ex-

EXAMPLE 4.

What is the Square Root of $\frac{1}{2}$?
The Log. of 1 is = 0·0000000
The Log. of 2 is = 0·3010300

Therefore, Log. of $\frac{1}{2}$ is = − 0·3010300
And the Half of it is = − 0·1505150; and the Number correſponding to this Negative Logarithm is $\frac{1}{1{\cdot}414214}$, which is $= \sqrt{\frac{1}{2}}$; where the Diviſion of 1 by 1·414214 is yet to be performed, (or ſome other Method taken) to obtain the Value of this Root. For obviating this Inconvenience there are ſeveral Methods. That derived from the following Conſiderations is the ſame, in effect, as the learned Dr. *Saunderſon*'s.

It is evident that $\sqrt{\frac{a}{b}}$ is $= \frac{10\sqrt{a}}{10\sqrt{b}} = \frac{100\sqrt{a}}{100\sqrt{b}} = \frac{1000\sqrt{a}}{1000\sqrt{b}}$, &c.

$$\text{And } \frac{10\sqrt{a}}{10\sqrt{b}} = \frac{\sqrt{100a}}{10\sqrt{b}} = \frac{1}{10} \times \sqrt{\frac{100a}{b}}$$

$$\frac{100\sqrt{a}}{100\sqrt{b}} = \frac{\sqrt{10000a}}{100\sqrt{b}} = \frac{1}{100} \times \sqrt{\frac{10000a}{b}}$$

$$\frac{1000\sqrt{a}}{1000\sqrt{b}} = \frac{\sqrt{1000000a}}{1000\sqrt{b}} = \frac{1}{1000} \times \sqrt{\frac{1000000a}{b}}$$

and univerſally, $\frac{n\sqrt{a}}{n\sqrt{b}} = \frac{1}{n}\sqrt{\frac{nna}{b}}$; where n is to be taken = 10, 100, 1000, 10000, &c. as the Caſe may require. Now, if we put $\frac{nna}{b} = c$, then will $\sqrt{\frac{nna}{b}} = \sqrt{c}$, and we ſhall have $\sqrt{\frac{a}{b}} = \frac{1}{n}\sqrt{\frac{nna}{b}} = \frac{\sqrt{c}}{n}$; and ſince n is to be ſo taken that c ſhall always be greater than 1, its Logarithm will be always affirmative; and conſequently, if we take the Number anſwering to that Logarithm, and divide it by n, (which will be no more than removing the decimal Point 1, 2, 3, or more Places towards the left Hand, according as n was taken $= 10$, 100, 1000, or more) we ſhall obtain the Square Root of $\frac{a}{b}$.

Now to apply this to the laſt Example, we have $a = 1$, and $b = 2$; and by taking $n = 10$, we ſhall have $c = \frac{100}{2} = 50$, and $\sqrt{\frac{1}{2}} = \frac{\sqrt{50}}{10} = \frac{7\cdot07107}{10} = 0\cdot707107$.

The Operation by Logarithms will ſtand thus:

The Logarithm of 100 is $= 2\cdot$
The Logarithm of 2 is $= 0\cdot3010300$

Their Diff. is the Log. of c $(= \frac{100}{2}) = 1\cdot6989700$
$\frac{1}{2}$ the laſt Logarithm is $= 0\cdot8494850$; and the Number belonging to it is $7\cdot07107 = \sqrt{c}$, which divided by 10 gives $0\cdot707107 = \sqrt{\frac{1}{2}}$.

EXAMPLE 5.

What is the Square Root of $\frac{3}{451}$?

Here n muſt be taken $= 100$, (becauſe $\frac{300}{451}$ is leſs than 1) and then $c = \frac{30000}{451}$.

Log. of 30000 is - $= 4\cdot4771213$
Log. of 451 - $= 2\cdot6541765$

Log. of c $(= \frac{30000}{451}) = 1\cdot8229448$
Log. $\sqrt{c} = 8\cdot15591$ is $= 0\cdot9114724$; and then $\frac{\sqrt{c}}{100}$ is $= 0\cdot08115591 = \sqrt{\frac{3}{451}}$.

EXAMPLE 6.

What is the Cube Root of 980?

The Log. of 980 is $= 2\cdot9912261$
$\frac{1}{3}$ of it is - $= 0\cdot9970754$; and the Number which anſwers to it is $9\cdot93288$, which is the Root required.

EXAMPLE 7.

What is the Cube Root of $\frac{1}{3}$?

This is a Caſe ſimilar to that of the Square Root of $\frac{1}{2}$; and the Inconvenience of negative Logarithms may be obviated by ſimilar Conſiderations.

For $\sqrt[3]{\frac{a}{b}} = \frac{10\sqrt[3]{a}}{10\sqrt[3]{b}} = \frac{\sqrt[3]{1000a}}{10\sqrt[3]{b}} = \frac{1}{10} \times \sqrt[3]{\frac{1000a}{b}}$;

and univerſally, $\sqrt[3]{\frac{a}{b}} = \frac{1}{n}\sqrt[3]{\frac{nnna}{b}}$; and by putting $\frac{nnna}{b} = c$, we have $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{c}}{n}$, where n is to be taken = 10, 100, 1000, &c. ſo that c may be greater than 1.

To anſwer the laſt Queſtion we need take n only = 10, and then c will be = $333\frac{1}{3}$, and we ſhall have $\sqrt[3]{\frac{1}{3}} = \frac{\sqrt[3]{333\frac{1}{3}}}{10}$. By Logarithms thus:

Log. of 1000 is = 3·
Log. of 3 is = 0·4771213

Log of c is = 2·5228787
$\frac{1}{3}$ of it is - = 0·8409596; and the Number is 6·93361 = $\sqrt[3]{c}$, and therefore 0·693361 = $\sqrt[3]{\frac{1}{3}}$.

EXAMPLE 8.

What is the Biquadrate Root of 17?

The Log. of 17 is = 1·2304489
$\frac{1}{4}$ of it is - = 0·3076122; and the Number belonging to it is 2·03054, which is the Root required.

Theſe Examples may be ſufficient to ſhow the Uſe of Logarithms in the Extraction of Roots, and the great Facility of that Method.

The Biquadrate, and higher Roots of Fractions are to be obtained by a Method ſimilar to that which was uſed for their Square and Cube Roots. And as the Algebraiſt cannot but be equal to theſe Things after ſeeing ſo many ſimilar Operations, I ſhall now end this Treatiſe (extended much beyond its firſt Limits) with a few Remarks.

It appears that the Ciphers added to the Numerator of the Fraction, for the ſeveral Roots, muſt be as follows:

For the Square Root, by 2s.
For the Cube Root, by 3s.
For the 4th Root, by 4s.
For the 5th Root, by 5s.

And that, as many Periods of Ciphers as were added, ſo many Places of Figures muſt the decimal Point be removed towards the left Hand, in the Root of *c*.

There are yet ſome Advantages to be obtained by the Uſe of Logarithms in the Extraction of Roots, which will be ſhown in my *Mathematical Eſſays*.

FINIS.

18 JY 64